A Multiphonic Reappraisal and the Alto Saxophone Concerto *Radial*

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ABSTRACT

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This dissertation analyzes the original composition *Radial*, which is scored for alto saxophone solo, small orchestra and live electronics. Multiphonics are a common feature on the aural surface of *Radial*. This analysis will show that alto saxophone multiphonics are also a primary structural element in the work, hierarchically organizing the timbres, harmonies, instrumental interactions and large-scale form of the score. Interestingly, no source suggests how multiphonics can be an independent organizational force. Numerous book length multiphonic catalogues for diverse instruments give fingerings for these sounds and describe them as harmonies so that they can be fitted into harmonic contexts, and a small but significant scientific literature on multiphonics discusses the acoustic principles underlying these sounds, but no document considers their independent structural potential. After providing a general account of multiphonics and their relation to harmonic and inharmonic sounds, this dissertation will propose an answer to that problem by drawing together concepts from American experimental music, spectralism and cognitive music theory, with *Radial* reviewed as an example of this method in action. Historical issues and a broad range of implications for this research will also be discussed.

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Dedication

This work is dedicated to my advisor, Fred Lerdahl, to my parents, John and Cecilia Moore, to my wife, Eirini Loulakaki-Moore, and to my daughter, Cecilia Loulakaki-Moore—inspirations and foundations all, seeing this work through to its conclusion.

1 Introduction

This dissertation explores the phenomenon of multiphonics and their deployment in the original composition *Radial*, a ten-minute work scored for alto saxophone solo, small orchestra and live electronics. Alto saxophone multiphonics are a conspicuous feature in the musical surface of *Radial*, but they also inform the large-scale structure of the composition. This question of how to establish *a functional approach to multiphonics* remains controversial, and the provisional approach given here was not fully established when the composition of *Radial* began. The speculative nature of the enterprise led to diverse sketches and prompted a wide survey of multiphonic catalogues for various instruments and scientific studies on the phenomenon. Ultimately, persistent descriptive shortcomings in the literature encouraged me to establish a new definition of multiphonics that separates these sounds into five categories based on acoustic properties; the model is laid out in Chapter 2. The literature on multiphonics (both practical and theoretical) is briefly surveyed in Chapter 3 but it is referred to throughout the text on a case-by-case basis. Chapter 4 takes *Radial* as an example of how this new model can inform a compositional practice. It begins noting the composition's basic affinity with works of American experimental music and spectral music and its more particular groundings in the acoustics of waveforms and musical instruments. It then proceeds by pursuing holistic descriptions of multiphonics that refer to their physical underpinnings, from which vectors of data arise that are related to key issues in Lerdahl's models of *Tonal Pitch Space* (2001) and timbral hierarchies (1987). What emerges is a specific and individual multiphonic case study—an “alto saxophone multiphonic space”—articulated in the first half of Chapter 4. The second half of the chapter explores in detail the structural role and development of the “space” in *Radial*. Contrasts to previous organizations and definitions of multiphonics are addressed throughout the chapter. Chapter 5 presents a brief retrospective assessment of earlier chapters and considers some more immediate implications of *Radial* and the new multiphonic models.

An underlying goal of the theoretical work surrounding *Radial*, and indeed of the composition itself, is to suggest how multiphonics can be integrated more fully into compositional aims. It is also germane, then, to point out how rich *the range of contexts* for this work might be. Appendix A provides a sociological and historical account of multiphonics that does just that. It also clarifies the historical narrative of multiphonics, which seems not to be published in a formulation accurate enough to merit citation. The version given in Appendix A is detailed and broad, though certainly not exhaustive. It sets a basis for further observations and encourages authors to move beyond histories of multiphonics relating only to one particular instrumental family or musical style. Finally Appendix B provides short discussions of several terms and concepts that often lead to hasty conclusions in multiphonic literatures. An up-to-date bibliography is given in the references section to support further research.

1.1 Practical and Theoretical Backgrounds of *Radial*

Radial is a highly theoretical work, and the present thesis underscores that point, but it is also part of a long dialogue I've had with multiphonics and the saxophone itself, the early components of which go back far enough that their influence on my work is hard clearly to assess and appreciate. These include extensive performances of multiphonics as a trombonist, compositions with multiphonic sounds, influential meetings and lessons with Al Grey, apprenticeships with important teachers like G. David Peters and Donald Rafael Garrett,[1] and above all the beginnings of a long friendship and working relation with Taimur Sullivan.

Taimur Sullivan, Professor of Saxophone at the University of North Carolina School of the Arts and a member of the PRISM Quartet, is the dedicatee of *Radial* and premiered the work with Columbia University's Manhattan Sinfonietta under the direction of Jeffrey Milarsky; he also premiered the other saxophone works discussed in this essay. Our working relation has been collaborative, with material and sketches tested at home, in the studio and, when needed, over long distance and transatlantic phone lines. This interaction is difficult to overestimate. (Section A.3 broadly argues that the intellectual and stylistic history of individual works is impoverished when these networks are not explored.) *Radial* is

[1] G. David Peters (later Director of the IUPUI School of Music) introduced me to brass sung multiphonics and the wider discography of multiphonics in jazz. Donald Rafael Garrett, an early associate of Muhal Richard Abrams, embodied that discography through recordings with John Coltrane, Dewey Redman, Rahsaan Roland Kirk, Archie Shepp and Kali Z. Fasteau. He was a gracious mentor with a unique practice of combining double bass and bass clarinet improvisations with vocal multiphonics.

designed to create, through its performances, a long conversation about the tie between its surface and structure. I would have surely hesitated in, and perhaps retreated from, poising its materials with the fine openness needed to launch this project, if I was not already deeply enmeshed in dialog with the premiere soloist.

After commencing undergraduate studies at the University of Illinois, with strong teachers like Herbert Brün, Erik Lund, Salvatore Martirano, Morgan Powell and Paul Zonn, who easily crossed lines between jazz and classical music, my exposure to multiphonics became too thick for me now to enumerate particular influences, except—of course—to note computer music studies through the lens of acoustics and psychoacoustics with James Beauchamp and masters-level composition studies with Anthony Braxton at Wesleyan University. (Later influences on the work are surely crucial, but these mostly come up directly in the argument of the text.)

In retrospect, it's surprising I haven't written more works with multiphonics. But composing with these sounds can be vexing. It can feel like being pulled into a bad crossword puzzle between compositional aims and material content. After a serious attempt to put multiphonics to wide use in *hiatus pitch* (1992) I pulled them from the composition entirely and mostly jettisoned these sounds until, almost by chance, I seized them again in *Black Box* (2001). The moment came from an elaborate electronic movement-sound interaction piece called *Overtime* (2000), which was programmed at the totally impractical location of Pier 21 along the western coast of Manhattan. (Beyond the typical swath of computers and devices, the work called on giant inflatable pillows and a custom-built sound isolation chamber that looked like a broad British phone booth.) In an effort to appease disappointed dancers, and find some workable solution for the venue, I decided to approximate the sonic effect of the performance in a composition for amplified saxophone by calling on the virtuosity of Taimur Sullivan and the sound world of multiphonics. Other works followed including *Forever Young* (2003) for two baritone saxophones and live electronics, which mixed classic spectral techniques (performed on "sound objects" derived from a wood and spring steel kalimba) and a variety of multiphonic and harmonic series-based sounds.

With *Radial* I decided to do conclusive work on alto saxophone multiphonics as a body of related sounds: the challenge moved my collaboration with Sullivan forward and offered a unique chance to bring recent and distinct compositional concerns together. Key concepts of spectralism including spectral stretching, compression and distortion, presented themselves as a tool to explain and deploy the stubbornly shifting harmonies of alto saxophone multiphonics. The formal concept of envelope dilation and transposition also appeared readymade for the task because each multiphonic is both a chord and a phrase waiting to happen. To be more specific, a typical multiphonic, which might be perceived as having four discrete notes, does not have a temporal character. It is a physical configuration on an instrument. Its material can be presented vertically, as a single sonority, or horizontally, as a series of single tones, dyads or triads.

American experimental music was also germane to the task. Murail (2005b) called Giacinto Scelsi the first "de-composer" because in compositions like *Anahit* (1983) he used one note to place a whole complex of sounds on the page—with additional materials intervening at the level of the component, breaking the synthetic recognition of partials and giving rise to wholly new entities. Such composition at the level of the wave is also characteristic of Alvin Lucier (Lucier 1995; Moore 2001)—with whom I studied at Wesleyan University—and other composers in the "spectral" school of American experimental music, including Phil Niblock, Pauline Oliveros and James Tenney. In *Draft* (1996), written for and premiered by Ensemble L'Itinéraire, I worked with de-composed states organized by intervals of one, two and three cents cued by a pitch track. Though it's a successful work, for which I am greatly indebted to Klaus Huber who offered enormous assistance during rehearsals for the premiere, I was ultimately reluctant to explore micro-level fusion of pitch and timbre any further by means of a rhythmically and gesturally confining pitch-track.

Multiphonic sounds, however, appear in already "de-composed" states. They embody diverse features that Scelsi and Lucier—and Murail—create through separate means. Like the music of Scelsi and the first spectral composers (arriving some two generations later), the sonorities of multiphonics are more harmonic and asymmetrical than the flat or "grid-like" spaces that often serve as departure points for instrumental compositions by Lucier or Niblock (or, in the chromatic context, Morton Feldman); but by extending from the acoustic properties of physical systems, they also align with the objective nature of Lucier's many resonator and reflector works beginning with *Chambers* (1968). Put more generally, multiphonics align with his concept of the resonator and the sound wave as a primary physical material. Rather than following musical developments bearing metaphorical relations to natural processes, multiphonics, if viewed more holistically from this latter angle, might reveal vectors of distortion embodied directly in the sounding mechanism itself. If so, could the strong timbres of these sounds help us build intuitions about these vectors? —about the "harmonic spaces" that encompass them?

To explore these intersections, it would be necessary to understand multiphonics not as a collection of individual chords but as a complete hierarchical body of sounds which extends from a single (complex) acoustic system. *Tonal Pitch Space* (Lerdahl 2001) provided a launching point for this description, with the theory of "Timbral Hierarchies" (Lerdahl 1987) helping to account for the exotic nature of these sounds. In short, taking up the subject of multiphonics once more presented a method of bringing together a wide range of influences embodied in my work and the depth of my studies. With these thoughts the composition of *Radial* began.

2 A Preliminary Definition of Multiphonics

2.1 Acoustic, Psychoacoustic and Stylistic Boundaries

Woodwind instruments, brass instruments, the human voice, percussion instruments, string instruments—for instance the individual strings of a violin, cello or piano, or in some cases the totality of the device—as well as a variety of natural and industrial phenomenon can all produce multiphonics. However, in each case, it is important that from a psychoacoustic perspective we associate the particular mechanism or thing with the production of single notes and that somehow it is made to produce two, or more, seemingly independent tones at the same time.

This list, while somewhat ungainly, addresses important caveats. Multiphonics are typically discussed in relation to a monophonic instrument, like a clarinet, and the unique properties of its reed. However, polyphonic instruments can produce multiphonics; our voices can produce multiphonics; instruments without cane reeds – like the flute – can produce multiphonics. Even bowed and struck instruments can produce multiphonics. Discussed independently these sounds can steal away under various terms: burr tones, half-valves, double harmonics, undertones, growls and wolf notes. But they all belong in the larger category multiphonics.

Perhaps the most troubling aspect in the list above is that the term multiphonic ultimately describes how we hear. Most objects produce more than one "sound," but we process them synthetically as a single note. A low trombone tone can carry 120 and more harmonics, but we usually hear them as the timbre of the first (or fundamental) harmonic. From the perspective of music cognition then, we can say something becomes a multiphonic when, for various reasons, we begin to sort these sounds out and identify them (analytically) as separate components.

While we can try to define multiphonics from a physical perspective that references perceptual boundaries, the traits we assign will inevitably blend into the life of conventional pitches and common noises. Any time we hear inharmonicity, hear out individual partials, or hear independent movement of amplitudes, do we want to call that sound a multiphonic? The answer is surely no. And so there is also a social or stylistic convention as to "when" is a multiphonic. The clarinet provides an excellent case as to "when" analytical listening (hearing out individual partials) shouldn't be taken as evidence for the presence of multiphonics. The clarinet's square-wave like timbre largely discards even-numbered harmonics, leaving notable gaps between the odd partials that help us to hear them individually. But when we observe these faint tones—usually the third and fifth harmonics—in a clarinet composition from the Romantic repertoire, it is surely better to think of the phenomenon we perceive as an instance of analytical listening alone. Nonetheless, we can also note the ambiguity and reflect upon its suggestive possibilities, and perhaps take them up later in a different context.[1]

[1] Elliott Carter's solo clarinet piece *GRA* (Carter 1994) takes advantage of precisely this ambiguity. In it large slow leaps of 12ths and beyond create chromatic passing motions between the harmonics of the first lower note and the fundamental of the second large leaping tone. Eventually the 12th f#4 to C#6 is heard several times as a harmonic series-based multiphonic giving us both leaping notes at once. Afterwards the 12th is heard again

With these cautions in place, let's hazard a physical definition of multiphonics and then discuss a few perceptual issues afterwards. The task is not problem-free. But if we have an understanding of how resonators combine to create musical sounds, we can do surprisingly well.

2.2 Energy, Resonant Systems, Multiresonant Systems, Coupled Resonators and Radiators.

To build strong intuitions about multiphonics, it is important to understand the physical basis of sound production in musical instruments. The generation of a musical tone requires at least one resonant system, which by definition is a system capable of storing kinetic energy and potential energy and passing energy back and forth between these two states. The Helmholtz resonator is a simple resonant system with one degree of freedom, meaning it supports resonance at only a single frequency. Most resonators, however, are multiresonant systems capable of supporting many frequencies at once (specifically we mean capable of supporting many frequencies *in a single position*, i.e. without lifting keyholes or changing string lengths, which is a still more complex scenario). Olson (1967) further stresses that most musical instruments have two or more multiresonant systems coupled together in circular chain of influence. The first resonator typically carries the

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melodically, descending from C#6 to f#4 allowing us to hear, or remember, the multiphonic once more embodied in the final f#4 and its third harmonic. So, in that case, the final f#4 (m. 95) is a conventional tone heard analytically as—or perhaps merely referencing—a multiphonic, which was stated explicitly three times, as three extended multiphonic long tones, in the preceding ten measures.

primary driving force and delivers it to the second resonator. The second resonator modifies the energy and sends back negative pressure pulses that influence modes of vibration in the first resonator while also passing energy forward, ultimately radiating sound into the air. (The second resonator may or may not achieve wide sound radiation on its own.)

Benade (1960) approaches the subject of coupled resonant systems differently. Rather than stressing the flow of energy he emphasizes damping relations between the resonators. In simple terms there are two varieties of resonators: heavily damped resonators that respond to many frequencies, but don't respond with great force; and lightly damped resonators that respond only very selectively to particular frequencies but with enormous force.[2]

As an example, let's use Benade's model to explain tone production in the primary coupled resonant system of the saxophone. Cane reeds are the heavily damped resonators of the saxophone—and most woodwind instruments. (Flutes are the exception, which rely on an air jet, sometimes referred to as an "air reed", to stimulate resonances in the instrumental body.) Reeds respond well to many impulses (frequencies) but when their

[2] Heavily (or highly) damped resonators possess high friction and short response times. Lightly (or lowly) damped resonators possess low friction and long response times. The resonance curves of heavily damped resonators, which loose their energy quickly, are low and broad. The resonance curves of lightly damped resonators, which possess long sustains, are steep and pointed; consequently they are sometimes called "sharp" resonance curves. Engineers refer to this relation between amplitude and frequency response as *Q*. The designation "sharp" has created some confusion: processors which impart "sharp" or "high" *Q* mimic the effects of lightly, or lowly, damped resonators. For more on *Q* see Rossing (1990).

resonant frequencies (typically above the conventional registral range of fundamentals) are excited they respond with only a moderate degree of additional force. This feature is crucial because for any pitch fingered on the instrument, the reed will have a wide range of similarly powered resonances available to drive the (mostly) harmonic resonances of the tube.

The body of the instrument is the lightly damped resonator and responds weakly to most impulses (frequencies), but when its resonant frequencies are excited it responds with enormous energy. In other words, the body of the instrument only responds to harmonic (or nearly harmonic) components of the reed and at the same time discourages and filters out non-harmonic components of the reed sound. The body of the saxophone, like all woodwinds, is so selective (so lightly damped) that the device must have keys allowing the performer, essentially, to change the tube length of the instrument as needed to create new pitches. (One implication of this is the highest large open key on the tube, which defines the active tube length, is also the primary radiator of the sound. The bell only serves as the primary radiator and ventilation point for the instrument when all keys are closed.) Ultimately the reed and body work together in a coupled resonant system mutually reinforcing a mostly harmonic body of oscillations.

Many instruments, however, contain more components than a simple two-part coupled resonant system. Some have additional resonators in the production chain. For this, we can take the example of the saxophone again, which like all wind instruments benefits from a third resonator — the performer’s vocal cavity. The vocal tract precedes the primary coupled resonant system and its energy follows the same vector. (That is it goes into the heavily damped resonator that provides the driving force and through to the lightly damped resonator that leads to the terminal radiation of the sound.) The vocal tract is therefore called an upstream resonance. It flexibly and continuously tunes behavior throughout the primary coupled resonant system of the saxophone. Detailed scientific study of its influence has only recently begun (Wilson 1996; Scavone 1997; Scavone 2006; Scavone, Lefebvre & da Silva 2008.)[3]

Finally, many instruments also have a special type of terminal resonator referred to as a radiator. Ideally, radiators have an extremely flat response (are very heavily damped) and a "weak" coupling such that they do not significantly alter resonant behavior higher up the chain. Instead they color the sound while projecting it forward. (This over simplifies matters, but we'll get more detailed shortly.)

The violin provides an excellent example of a musical instrument with a primary coupled resonant system, additional resonators and a distinct terminal radiator. The primary coupled resonant system is formed between the bow and string contact point (a motion of pulling and dropping which creates a rich sawtooth waveform)[4] and the harmonic

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[3] The great flexibility of the vocal tract makes it more difficult to describe. Generally speaking the vocal tract is highly damped. However, unlike the reed, its resonant frequencies are in a musically pertinent frequency range and have crucial noticeable effects. Finally damping is lighter in the lower frequency range and becomes heavier with increase of frequency.

[4] The sawtooth waveform described above is simple, idealized "Helmholtz motion." For a fuller contemporary account of waveforms produced between string and bow, including "double-slip" and "multiple-flyback" motions, see Fletcher (1998). For an astonishing account of Helmholtz discovering this basic motion by attaching a microscope to a tuning fork set in sympathetic motion by the specimen see Helmholtz (1863/1885).

resonances of the string itself. Other open strings provide a few more lightly damped resonators, which respond to their various natural frequencies.[5] And the violin body is the radiator, the crucial terminal resonator for power.[6]

2.3 Five Basic Types of Multiphonics.

Drawing on this straightforward discussion of sound production in musical instruments we can isolate five basic types of multiphonics, which in some cases may overlap in a single sonority: 1) collateral multiphonics, 2) harmonic series-based multiphonics, 3) radiated multiphonics, 4) multi-driver multiphonics, and 5) non-coincident multiphonics.

2.3.1 Collateral multiphonics.

Collateral multiphonics are produced by ambiguities in the second resonator (the lightly damped resonator) of a simple coupled resonant system. Therefore they are associated mostly with keyed instruments that allow for a wide variety of long fingerings or

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[5] If they are fingered, their natural lightly damped frequencies shift with the fingering—whether or not they are played. From this perspective "artificial harmonic" is surely an unfortunate choice of terms.

[6] In a bigger study it would be appropriate to consider its plates and elements separately, rather than to simply refer to the resonance of body itself. For more on this see Wang (1999).

simultaneous alternate column lengths (see Figure A.1.) One or more (large or small) keyholes opened above the point of ventilation allow the body to support two or more inharmonically related fundamental resonances (i.e. air column lengths). It is possible to create the same phenomenon by closing one more (large or small) keyholes below the point of full ventilation. In this case increased resistance downstream of the ventilation point allows the air column to dissipate less rapidly, thereby extending further down the tube.[7] This may allow access to one or more ventilation points, once again creating stable multiple column lengths. The two processes can also be combined giving the point of full ventilation both venting above and resistance below. More generally, this kind of collateral multiphonic might be thought of as providing two or more fingerings at the same time. The body, however, still remains lightly damped so in spite of the unusual quality of the sound the multiphonic still exhibits tightly controlled behavior.[8] Crucially the reed, being heavily damped, is able to support both of these fundamentals at the same time. Typically, the

[7] Eventually enough resistance added downstream, or enough ventilation added upstream, will likely shift the primary ventilation point, fundamentally altering the column lengths at play. If so, it is surely pertinent to re-conceive the fingering according to the simplest underlying fingering (and the notes heard.) For instance, if a point of full ventilation has one closed key above it and three consecutive ventilation points open above that, its functionality is likely gone. Its influence, if it has any, will be as a post-resistance opening of a higher fingering.

[8] The situation regards damping warrants further investigation. Some multiphonics are muted where others seem to respond with even greater force than conventional tones.

performer widens the focus of the vocal resonance to encompass the needs of both inharmonic tones.[9]

Though the effect has less variety (many woodwinds have upwards of 500 separate collateral multiphonics) and different qualitative response, trumpets can also create collateral multiphonics (body support of two simultaneous inharmonic air column lengths) through half-valuing and a trombonist can do the same through an F attachment. String instruments can create collateral multiphonics by bowing with great pressure. This causes a rapid alternation of two string lengths and tensions to manifest as the string both stretches and snaps back under duress.

It is clear the violin example must be some kind of rapid trill. It is unclear if a related alternation underlies the other collateral multiphonics described (they might simply co-exist). If there is such an alteration, as Benade (1976) proposed, it still remains to be stated what temporal characteristics it has.

The quality of collateral multiphonics are deeply influenced by design factors in particular instrumental families. Cylindrical woodwinds—the flutes and clarinets—create

[9] A further practical detail is important to consider. Not all long fingerings will produce collateral multiphonics (though most will). It is possible for a small key to be opened higher than the point of full ventilation, which coincides with a node—low pressure point—in the air column. In this case the change may not significantly alter ongoing harmonic oscillations. Similarly, closing holes far below the point of full ventilation may not add enough resistance to significantly alter activity higher up the bore. In short, it's always pertinent to listen before categorically assuming you've switched from a fully vented fingering to a functional long fingering.

deeper and more varied stable subsets of collateral multiphonics (i.e. sonorities that can be systematically transposed across portions of the instrumental register.) Limits to these transpositions often do not reveal deep structural properties so much as mere inconveniences of traditionally designed keywork—from makers who did not anticipate the need for such fingerings to be reproduced up and down the bore of the instrument. The newly designed Kingma flute (Shiung 2008) allows similar fingering permutations from each tone whole.[10] Consequently it plays transpositions of particular multiphonics whose holes are “on” the Boehm flute but simply can’t be lifted at once on that instrument. At the same time it is important to stress that not all collateral multiphonics on the flute are transposable. Many collateral multiphonics, often described as undertones, provide a variety of higher pitches with a lower C5 or C#5 (Levine 2002); and these sonorities can’t be shifted around. Clearly the 2nd harmonic of the instrument’s fundamental resonance, which divides the tube in half, has broad power that arises in many contexts and can be retained even when distorted by a half step. Thus the unique physiognomy of the instrument (see Section A.2.3) combined with the stability of its “air reed” allows for multiphonics that are impressively transposable (if not perfectly symmetrical), but it also creates multiphonics bound by totally unique harmonic (and physical) centricities.

The clarinet, owing to its cylindrical bore, shares many of these properties with the flute, but it has important divergences. The clarinet, with its reed made of fibrous cane, can

[10] Within obvious limits: a four tone whole pattern can’t be played from the third-to-last tone whole—which only has two holes above it!

produce deeper subsets of multiphonics rich in higher partials (Rehfeldt 1977).[11] Its collateral multiphonics are also less transposable than such sonorities on the flute (though nonetheless remarkable relative to most woodwinds) because the sizes and linear dispositions of the clarinet's keys are more diverse giving rise to greater asymmetric shifts of chord and color (see Section A.2.3).

The collateral multiphonics of conical wind instruments—bassoons, oboes, and saxophones—rarely produce substantial subsets of chords in clear transpositions. Instead two different streams of distortions mark their multiphonics. The first stream of distortion is a global tendency for chord types to change—though not always predictably—across register (see Section 4.2.4). The second stream of distortion is more local. Here we note chords in particular positions that appear in near-unison chains of distorted multiples. Especially on the double reed oboes and bassoons, but also on the saxophone, we find such streams of multiphonics that look quite similar when notated as chords, but have strong progressive timbral alterations sounding like one entity gradually reaching a breaking point. Sonorities on either side of this continuum are hardly interchangeable even though they might be rough equivalents as pitch sets.[12]

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[11] Tellingly, Benade (1996) reports removing the mouthpiece from the body of a clarinet and precisely fitting a flute head joint onto the body. The hybrid instrument displayed an uncanny resemblance to the tone production of the flute.

[12] For instance, see Veal (1994) multiphonic nos. 329—332 and 341—344.

The point of designating these sounds "collateral" is that they exist side-by-side but descend in parallel fashion, or along the same vector. Consequently we have inharmonic driving tones, but the nonlinear distortion that arises can be—relative to one particular tone—either harmonic or inharmonic. Very briefly, given two inharmonically related tones $f1$ and $f2$, nonlinear distortion can produce another tone ($f1 + f2$) that will be inharmonic relative to both drivers, but it can also produce frequencies such as $2f1$ or $2f2$, which are clearly harmonic relative to one driver or the other (see Section B.2).

Although collateral multiphonics are associated with keyed wind instruments (saxophones, flutes, oboes or clarinets) their sonorities blend well with inharmonic percussion instruments like church bells, tam-tams and steel drums. These instruments also have deeply inharmonic partials, but there are important structural differences between the two groups. The resonances are inharmonic metals listed above are closely related to the modes of vibration found in plates. Rather than having inharmonic resonances that travel along (or alternate on) the same vector, these instruments are two-dimensional resonators and therefore have nodal lines instead of nodal points. These lines cross over each other (i.e. have transverse relations) and prohibit the development of harmonic partials (Rossing 1990). However, on these instruments one or more higher partials is often hammered or sculpted into harmonicity with the fundamental resonance, creating a mixture of harmonic and inharmonic frequencies relative to the lowest mode of resonance.[13] The steel drum is a

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[13] Such a process is also used on the vibraphone, where the underside of each bar is sculpted to bring a higher partial into alignment with the fourth harmonic of the fundamental. The underlying resonant patterns of the vibraphone are, however, different from those of

particularly striking case in this regard. Each pitch on the steel drum has at least one higher overtone hammered to match the 2nd harmonic of its fundamental. But the final spectrum of a steel drum tone arises from the primary note (and its overtones), the partials of neighboring pitches ringing sympathetically with the initial complex, and the nonlinear resonance that arises from these components. (Because the components are harmonic and inharmonic the nonlinear distortion is also harmonic and inharmonic.) When a steel drum note is hit at a normal playing volume, resonances are contributed from across the entire surface of the instrument (Fletcher & Rossing 1998). The sound of a steel drum can therefore come quite close to that of a rich collateral multiphonic.

The uneven or independent amplitude developments between partials on these instruments, which often unfold over long time intervals, also liken them to the world of multiphonic sounds. Whether or not we call inharmonic plate-like metals "multiphonic," we can appreciate and make use of their shared structural and acoustic properties

2.3.2 Harmonic series-based multiphonics.

Harmonic series-based multiphonics are usually produced by clearly defined second resonators (the lightly damped resonator) in a simple coupled resonant system. These multiphonics are found in all the woodwinds and brasswinds. Keyed woodwind instruments produce them through fully vented fingerings (see Figure A.2), such that all holes are closed

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vibrating plates. The vibration of bars is closer to that of strings, but their restoring force comes from the stiffness of the material, rather than from tension. Consequently, bars also have inharmonic partials, with remarkably high ratios between overtones: 1 : 2.76 : 5.40 : 8.93, etc. (Rossing 1990).

until the bell, or the arrival of the first open key, after which point all keys are open. Thus there is an unambiguous length from the driving resonator to the point of full ventilation. Interestingly, short of half-valving, or the use of triggers, this is the default condition of all brass instruments. They deal with one air column length at a time. Therefore most brass multiphonics are harmonic series-based multiphonics. To summarize harmonic series-based multiphonics, let us state: the lightly damped body supports (mostly) harmonic overtones based on one unambiguous air column length. The reed or lips, being heavily damped, are able to support a group of partials closely associated with the air column. And, once again, the performer typically widens the focus of the vocal resonance to encompass the needs of the particular target group.

Some of these multiphonics are smooth—they *are indeed harmonic*. These aren't the norm, but they certainly exist as a category, especially on flutes and clarinets. Other harmonic series-based multiphonics are quite rough. In part this is because many conventional instruments, from piano to trumpet to saxophone, have inharmonic spectra in the *standard tones* we routinely hear as single pitches. Rather than being strictly harmonic oscillations, such tones are better thought of as "oscillation regimes" (Benade 1976; Loy 2007). They are comprised of harmonic and inharmonic partials that nonetheless function cooperatively creating a stable group of oscillations. When a performer uses her vocal tract to revoice such a sonority, turning it into a harmonic series-based multiphonic, this inharmonicity often becomes more pronounced. Many conventional tones are also comprised of "would-be-oscillations-regimes," which are so tight they mode-lock when played as conventional pitches (i.e. the oscillation regime literally tunes up or falls into place producing the spectra of a harmonic oscillation.) By varying degrees, the instrument or the instrumentalist can be responsible for this. But if the performer uses vocal resonances to create unusual distributions of amplitude energy amongst the harmonic components, the mode-locking will break with it, bringing in mistuned harmonics together with the exotic amplitude energies of the harmonic series-based multiphonic. “Harmonic” and “oscillation regime” multiphonics are not antithetical. A noisy oscillation regime multiphonic with six or more partials can create a strikingly clear residual pitch (Veal 1994). And if a performer is aware of what’s under her fingers (or fingerings), she may find she has significant top-down control over tuning, creating transitions between one state and the other. Thus *harmonic and inharmonic* multiphonics that interact with one stable air column length are best thought of as “harmonic-series based multiphonics.” Basic design factors of the air column (conical or cylindrical, opened at one or two ends) and the driver (air reed, cane reed or double reed) also have predictable and crucial correlations in the qualities of these sounds.

Harmonic series-based multiphonics on brass instruments have a weaker coupling between the driving resonator (lips) and second resonator (tube). The driving force is more responsible for “finding” frequencies the second resonator will react to. Consequently, the experience of producing them is also different. Rather than moving smoothly from pitch to multiphonic the novice performer (especially) often experiences an abrupt shift to a rough condition requiring a movement back to the harmonic basis of the sound. Significant practice is required to repeat sonorities with clearly similar onsets and frequency distributions. These sounds are generally noisy, though in varying degrees. Nonetheless, stability provided by the body will always be harmonic series-based.

Related effects in string instruments can be created by bowing on the nodes of a string producing two or more clear notes from the harmonic series at the same time. Of course extreme sul ponticello is a plain method of producing pronounced overtones groups, but the enveloped characteristics of these sounds usually lead us to judge them synthetically. More pertinently, harmonic series-based multiphonics are encountered in a wide variety of world music contexts ranging from throat singing, to overtone flutes, mouth harps and mouth resonated musical bows. However the acoustic flow diagram in a few of these cases is slightly different from the example given at the top of the section. (For more on these instruments see Sections A.1 and A.1.1.)

Two final points must be made on the distinction between collateral and harmonic series-based multiphonics. First collateral multiphonics will always be made on top of or in addition to harmonic series-based multiphonics. Therefore there is a progressive, though sometimes surprising, relation between them; and distortion in the set of harmonic series-based multiphonics can typically be taken as a baseline for harmonicity in collateral multiphonics. Consider the case of the saxophone. Harmonic series-based multiphonics usually produce subtle stretchings in higher partials, beginning with the second harmonic. This is also true for the "prototype" collateral saxophone multiphonic, which effectively characterizes over two thirds of the instrument's collateral multiphonics (see Section 4.1.2). Even the small set of saxophone multiphonics with members that do not relate to the harmonic series — for instance a bass position tone and a minor third above it — still tend to interact with harmonic series-based multiphonics. For instance, low minor thirds and seconds in saxophone multiphonics usually seem to have two explanations. They appear to be distortion products between bass position tones and upper partials that are approximately related to the harmonic series, and they appear to be enriched by register specific collateral ventilations. Only around fifteen collateral ventilations actually create two independent columns a third or second apart, that sound as dyads with no salient trace of a background harmonic series-based multiphonic. (For saxophone multiphonics with low small intervals see the first half of Figure 4.11.)

The second issue is more practical. It would be convenient to describe collateral multiphonics as arising from special fingerings and harmonic series-based multiphonics as arising from standard fingerings, but this is not the case. Special fingerings and standard fingerings are historical categories, not scientific categories. Without specific cause to do so, performers won't typically dwell on whether or not a standard fingering is a long fingering (see Figure A.1) and therefore most likely produces a collateral multiphonic rather than a harmonic series-based multiphonic. Further most keyed instruments have keys in default positions that are either open *or* closed. Consequently musicians more immediately think about pressing a piece of keywork, which may or may not mean opening or closing a hole. Fully vented fingerings and long fingerings are musical terms, but they are less familiar to musicians than the body of standard fingerings and the reality of pressing and releasing keys.

However, familiar or not, they are very important concepts for multiphonics because they have clear acoustic implications.

2.3.3 Radiated multiphonics.

Radiated multiphonics are produced by extremely weak couplings between the first and second resonator (the lightly damped resonator), such that, in spite of its resonant characteristics, the second resonator essentially becomes a radiator of the sound emanating from the first resonator. These sounds are usually rich and flexible, and can be quite noisy. The didjeridu is a well-known case. (For a detailed discussion see A.1.) Its tube, with large diameter relative to its length, does not enforce or impart a harmonic organization based on its dimensions. The spectra and the fundamental the performer chooses are largely determined by the lips (first resonator and driving force) and the vocal tract (upstream resonances). The tube simply colors the sound as it radiates it and provides noticeable pressure to the performers lips (relative to lip buzzing in open air); beyond this it gives little influence on vibrational modes higher up the chain. (For more on the didjeridu see Sections A.1 and A.1.1.) As noted above, brass harmonic series-based multiphonics—through their weak couplings—are a step in this direction. Consequently, it is possible for brass players to play full-blown radiated multiphonics. The player uses the embouchure and vocal tract shape needed for a harmonic series based-multiphonic, but blows it into an inharmonically related tube length. The great damping power of the tube when away from resonant frequencies will make the sound warble, but if the performers tightly fixes the limits of the embouchure and blows with great force it is possible to dominate the tube and turn it into a radiator of a relatively stable sound.

These sounds can be practiced and repeated with great accuracy but they are hard to pass on from performer to performer. Veal (1994) notates over 150 collateral oboe multiphonics with a “Z” factor indicating the use of pressure from the teeth on the reed. This rigid interference is best viewed as a form of radiation because it “overcomes” the natural resonances—harmonic or otherwise—of the system. Christian Hommel of Ensemble Modern (personal communication, January 5, 2010) warns these multiphonics are unplayable by most performers. Lacking any feedback or “control,” from the body or reed, the performer is left biting at the cane in various ways to try to isolate one of many possible complex sounds.

2.3.4 Multi-driver multiphonics.

Multi-driver multiphonics are produced by two or more driving forces following the same vector into a lightly damped resonator (usually thought of as the second resonator), which either radiates or leads to the radiator of the coupled resonant system. These are most often associated with “sung multiphonics” where wind instrumentalists use their sung voice as a driving resonator that follows through the vocal tract (now no longer being strictly “upstream”) into the second driving resonator (lips or reed) finally passing both sounds into the body of the instrument. However, there are other means of producing these sounds. Brass players, for instance, performer multi-driver multiphonics by splitting their embouchure, having one side of their lips vibrate at one frequency and the other side vibrate at another. One can imagine further possibilities.

The effects of multi-driver multiphonics might seem too diverse to enumerate, but there is an important detail to note in this context. It is well known that when musicians sing and play at the same time additional combination tones are created. However, there are issues in the response that are rarely explained. Consider this scenario: when a brass performer plays a traditional note and adds a sung note we have two drivers, but the choice of sung note will, additionally, make the sonority either a harmonic series based-multiphonic or, at least in part, a radiated multiphonic. If the sung note is inharmonic relative to the (unambiguous) tube length, the tube won't support it and it won't support many distortion products associated with it. The effect is surely noisy, but the tube is indiscriminately passing the noise and sound along. When the sung note is *harmonic* relative to some component in the sound or supported by the tube, the coupling will strengthen, creating something larger than the sum of its parts. Obliquely, most lists of sung chordal multiphonics confirm this (Davidson 2005). Authors rarely have had confidence to notate the content of sung chords that were not simple harmonically related sounds.[14] In short, what makes an inharmonically

[14] The harmonic relation doesn't have to correspond to the fundamental of the played note. It simply has to be harmonic relative to something in the played sonority, or supported by the tube. For instance imagine a 200 Hz tone played on an instrument with harmonics 400 Hz, 600 Hz, 800 Hz and 1000 Hz One could sing 300 Hz (a fifth above the fundamental, which is not a harmonic of the tube length) and this sound will interact, through nonlinear distortion, with the 1000 Hz and 200 Hz tones to create combination tones at 100 Hz and 500 Hz. These sounds are harmonic to elements already sounding in the sonority. Such sounds find reinforcement, but the lightly damped tube won't readily create powerful combination tones relative to a fundamental not supported by any of its resonances.

related "sung multiphonic" so different from a collateral multiphonic is that the lightly damped tube and driver of the collateral multiphonic supports both fundamentals, and the resonant system supports (though does not demand) nonlinear resonance. But the inharmonic "sung multiphonic" has one resonated sound and one radiated sound. Now that spectral analysis has revealed more clearly the inharmonicity of conventional tones—the presence of oscillation regimes—there may be a way to extend the gamut of stable sung chords into more interesting harmonies by coordinating sung elements with naturally inharmonic resonances. Certainly these criteria suggest a wide range of strategies for combining sung and collateral sounds.

2.3.5 Non-coincident multiphonics.

Non-coincident multiphonics are perhaps the strangest case here. They are produced by powerful downstream resonances that work their way up into the simple coupled resonant system, disturbing waveform development in the positions where they are being formed into stable oscillations. The best-known example comes from the violin. As we have said, ideally, its body is so heavily damped that it responds to all frequencies with near equal power and simply colors and projects the sound it receives. However, no radiator is ideally flat. Most violins have at least one frequency that is lightly damped, and when it is excited it responds with such force that it markedly alters vibrations higher up the chain on the strings and the fingerboard itself.

These sounds are called wolfs and they are certainly multiphonics. Unusually, however, they arise from a struggle between two non-coincident resonances (meaning the conflicting resonances arrive at a central point from different vectors). In the case of the violin, the wolf is a struggle between resonances arriving on strings from the driving force of the bow and the radiation of the body respectively. Instances of these are harder to find, but not when we step into the domain of electronics, where piezo microphones and piezo drivers can be laid on instruments in any fashion you can imagine. Perhaps this category of "disturbances between things" could also be extended more generally to include the interference effects of multiphones.[15] (For more on multiphones see Section A.1.)

2.4 General Remarks.

It might be noted this discussion hasn't dwelled on the role or theory of combination tones in multiphonics. The reason for this is their explanatory force is generally overstated. It is the conical woodwind collateral multiphonics that made the "frequency modulation patterns" of multiphonics famous. The arithmetic of combination tones is often addressed in relation to amplitude modulation and frequency modulation as a way of explaining multiphonics (Benade 1976; Backus 1977; Veal 1994). When looking at an oboe or bassoon multiphonic, driven by the great feeding force of two heavily damped cane reeds slapping against each other, it is indeed helpful to observe that 30 or more tones bear one primary

[15] Certainly the multiphone performances of Rahsaan Roland Kirk (1965/1967) have an immediate aural affinity with multiphonic performances themselves.

mathematical relation! However, if the collateral interaction is smooth enough, it is possible that little or no high profile nonlinear distortion will arise. Flutes, clarinets and saxophones (although to a smaller degree) all produce significant bodies of collateral multiphonics that are convincingly described as two note sonorities; and many of their three-note sonorities, which could be explained by the math of combination tones, have a deeper physical reality as primary air columns. This is evident because in such cases any of the three notes can be played singly going into, or coming out of, the multiphonic. Therefore one note is not being simply "produced" by the other two. (There are certainly are cases of three note multiphonics where a middle or outer term is a difference tone, but these are not a clear majority amongst three note multiphonics. For examples of both varieties see Figures 4.2 and 4.3. Pitches beamed as eighths and sixteenths show the notes of the sonority that can be played separately.) In short, explanations of multiphonics based on frequency modulation patterns fail to account for the harmonic (or physical) relations between the various driving tones—which are the first factors in a multiphonic and in many cases may be all that is present in the particular sound under consideration.

Nonetheless combination tones in multiphonics are important, and problematic confusion still lingers about their nature—confusions both about their presence in nonlinear distortion and linear constructive and destructive interference, and about the roles our ears, musical instruments and electronic technologies play in producing them. In short, combination tones can be observed with objective power inside or outside of the ear when they arise through the nonlinear distortion of a particular physical or analog electronic system, such as the ear, a musical instrument or an analog ring modulator. (They also have independent energy when presented by digital models of one of these systems.) Difference tones can appear vividly in the open air when two *separate* instruments each play one of the two driving tones. These difference tones arise through linear constructive and destructive interference and therefore have no physical power of their own. These tones are not observed by frequency analysis (this shows which frequencies contribute how much of the total power present in the signal) but our ears can hear these tones through repetition rate and envelope repetition rate analysis (Plack et al. 2005). What is not commonly appreciated is that this latter process is a central component of our pitch perception generally. It is a mistake to suggest hearing these tones requires extra processing. Otoacoustic versions of these tones sometimes arise as a redundancy in the system, but they are not required for us to hear these tones. It has been argued that a special process must arise above the level of the cochlea because we can hear these tones when the stimulus is presented binaurally (Houtsma 1971). However it is now assumed that data is routinely combined and reevaluated at different junctures in the auditory cortex. If the mind performs a second holistic repetition rate analysis, it is once again unclear that any *unique* process is devoted to interpreting repetition rates without unique power and providing us with a conscious interpretation of them. It is generally recognized that combination tones arise from nonlinearities in electronic instruments, acoustic musical instruments, and the ear. The basic math of combination tones is easily demonstrated through the example of amplitude modulation. However, the complexity of combination tones in biological and complex physical systems is much more difficult to describe. The response of the ear, for instance, is highly asymmetrical and involves separate layers of nonlinear activity including passive pre-filtering, active amplification, and cyclical two-term distortion product generation. Descriptions of the later cases must look at *what happens to* the basic mathematics of combinations tones if they are to be functional accounts. (See Figures 4.2 and 4.3.) Finally it is often not appreciated that nonlinear distortion itself can be either harmonic or nonharmonic—or both. (For more detail on these points see Appendix B: Key Terms and Issues for the Study of Multiphonics.)

Helmholtz's (1863/1885) defined noise and tone this way, "we perceive that generally, a noise is accompanied by a rapid alternation of different kinds of sensations of sound…on the other hand, a musical tone strikes the ear as a perfectly undisturbed, uniform sound which remains unaltered as long as it exists, and it presents no alternation of various kinds of constituents" (page 7). Today we understand musical tones to exhibit something closer to "coordinated change"[16] where Helmholtz instead speaks of "uniform" and "unaltered" sound. What interests us though, is his concept of alternation. Multiphonics are a step towards noise, they exhibit many interior noisy alternations, but their inharmonicity results from two or more *stable alternates or foci*. From a timbral and harmonic perspective multiphonics rest in a transition between noise and tone, and that is where their fascination lies.

[16] For instance, we know that musical sounds begin with noisy attacks and that the amplitudes of harmonics rise and fall in an integrated but individual fashion across the envelope of particular sound.

3 Theoretical and Practical Literature on Multiphonics

3.1 A Multiphonic Theoretical Retrospective

The prominent Italian clarinetist Temistocle Pace (1898—1944) tells us the first individual to theorize the production of multiple sonorities was one Antonio Ferrannini of the Conservatory of San Pietro a Majella in Naples (Pace 1943), but dates and details for this study are hard to come by. More clearly, Hermann von Helmholtz (1821—1894) laid much of the technical foundation needed to study multiphonics in his magnum opus *On the Sensations of Tone* (Helmholtz 1863/1885). Though he doesn't detail them explicitly, he comes tantalizingly close to doing so in his extended discussion of combination tones. In the first of six chapters devoted to combination tones, Helmholtz states that instruments can produce difference tones and summation tones that are scientifically observable in an open room and thus fully independent of the inner ear: "the condition for their generation is that the same mass of air [inside the chest of the instrument] should be violently agitated by two simple tones simultaneously."

Here Helmholtz describes masses of air contained in the wind chests of harmoniums and polyphonic sirens. He is clear the tones must be inside the same resonator, i.e. "the chest of the instrument", but he is one step away from inferring—or explicitly stating—the body must provide, or support, nonlinear resonances at those frequencies to give them observable unique energy. With that step he would have given a near perfect description of multiphonics and backed up his claim for observable energies in combination tones.

However his attention was absorbed in using combination tones to establish a general theory of harmony. Ultimately, not being perfectly clear on this point (both in describing the scene of his experiment and its precise theoretical implications), Helmholtz's tests weren't easily replicated nor were they appropriately interpreted. Ellis was fascinated by combination tones (1879—80) but on this point he sided with more skeptical studies from Bosanquet (1867—77; 1880). His position on the matter went into his generally excellent editorial remarks accompanying his English translation of Helmholtz's work. (Where they still appear in reprints today.) Consequently, deep skepticism emerged about the existence of sum tones, and difference tones were assumed to exist only within the inner ear, taking on the misleading designation "subjective tones" (Gough & Robison 1920; Meyer 1957; Rayleigh 1894/1945). This interpretation persisted well into the 1960s (Benade 1960) and beyond; it still pops up today in a variety of important music texts, dissertations, and reliable encyclopedic sources—such as Encyclopedia Britannica (as recently as 2010). Therefore, it was with clear-headedness and a measure of hesitation, that the acoustics of vocal multiphonics on brass instruments were indeed discussed in two articles by Kirby (1925) and Blandford (1926). Unfortunately, the impacts of these articles remained local.

3.2 Multiphonic Theoretical Studies After 1950.

Physicists John Backus (1911—1988) and Arthur H. Benade (1925—1987) were among the first modern acousticians to publish original work on multiphonics. In 1960 Arthur Benade summarized current physical understandings of music and musical instruments in his detailed monograph *Horns, Strings and Harmony* (Benade 1960), which is still valuable today for the conceptual clarity of its prose. The book discusses combination tones in detail but it assumes these are "subjective" sounds and makes no mention of multiphonics at all. Thus, Benade candidly admits he was not prepared to give an answer when an engineering friend wrote to him in the early 1960s inquiring how John Coltrane produced "certain sounds with multiple tones on his tenor saxophone." After the publication of Bruno Bartolozzi's *New Sounds for Woodwind* (1967) Benade decided it was time for investigations to begin (Benade 1976).

In the following years Benade (1976) and Backus (1978) published important work confirming that combination tones around two or more inharmonic simple tones were responsible for the spectral aspects of multiphonic sonorities in woodwinds; and that these frequencies (the driving tones and their combination tones) registered unique energies detected in open air traveling from the instrument towards the listener. Therefore these sounds were not necessarily, or by definition, subjective. Any multiphonic (or combination tone) theoretical work before this date is suspect.

Further it was emphasized that combination tones with discrete energy are produced by the nonlinear resonance of instruments whether or not the simple driving tones are *harmonic or inharmonic*. Thus combination tones were found also to be at work in conventional *harmonic* instrumental tones, but because they only replicate or add to the natural harmonics already present (Benade 1976), they had not been previously noticed or emphasized.[1]

It was not, however, until the publication of *The Techniques of Oboe Playing* (Veal & Mankopf 1994) that Backus's model was systematically tested and seen to explain an impressive range of partials found in oboe and English horn multiphonics. This remains one of the most thorough scientific studies to date. Important work continues to be done but it is typically aesthetic or lexical in character.

In one interesting case, which is ultimately aesthetic rather physical, Gottfried (2009) discusses clarifying notations for multiphonics based on sideband ratios. Briefly he reviews how, for many multiphonics, the individual tones of a multiphonic can be reduced to sideband ratios (frequency or amplitude modulation patterns) around two or more central driving tones (Backus 1978). He then suggests relating the central driving tones to a low common fundamental, much like Terhardt's (1982) measure for virtual pitch. However, many multiphonic driving tones are indeed inharmonically related. This means, from a scientific perspective, they are on the pathway to noise. Through combination tones of consecutively higher orders ($f1 - f2$, $2f2 - f1$, $2f2 + f1$, $f2 - 2f1$, $f2 + 2f1$, $2f2 - 2f1$, $2f2 + 2f1$ etc.), inharmonically related driving tones can generate all frequencies, while harmonically

[1] "Add to" is the pertinent qualifier in the case of hearing and reproducing the square wave, because $3f - 1f = 2f$. This would apply to our hearing of the more "square wave like" tones of the clarinet as well. However, while our ear might provide some even harmonic nonlinear distortion when hearing these tones, the single ventilation point cylindrical bore of clarinet can suppress distortion products and harmonics alike, so this example should not be taken to imply that nonlinear distortion freely fills in the gaps of the clarinet itself, nonetheless it surely contributes to its softer spectral profile.

related pairs can only generate tones in that particular harmonic series. (Jeans 1937; Knuth 1998). Relating the drivers to an ideal low pitch confuses one of their most basic properties. Musically, of course, we are interested in the continuum from harmony to noise and Gottfried's system or Terhardt's virtual pitch could be used as a rough measure of a multiphonic's dissonance, or to help liken it to other harmonic sounds, but it is not a more accurate system. And it is unclear if it is useful to introduce relatively unfamiliar accidentals derived from just intonation to present an idealized notation of an inharmonic sound.[2] This idealization creates further problems if we take another step away from abstraction. Previously (Section 2.3.2) we noted that common resonating systems, from pianos to trumpets to saxophones, will at times present the harmonic series in a distorted form—as inharmonic oscillation regimes—even in the production of conventional tones. The same instruments can also present frequency modulation patterns built around two central driving tones in distorted form. Even if the driving tones were harmonic, relative to a low fundamental, the frequency modulation patterns they create might be answered to, by the instrument itself, with an approximating inharmonic oscillation regime (See Figure 4.3). And those distortions wouldn't be at all incidental, rather they would be a fingerprint of the instrument itself—revealing more data about the same forces and logics that underpin an instrument's brilliant registers, muted tones and characteristic articulative voice. It would be a mistake to factor these distortions out.

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[2] Howell (1974) took this approach in his seminal work *The Avant-garde Flute: A Handbook for Composers and Flutists*, but it's the text's most limiting feature (Shiung 2008) because high-term just intonation is unfamiliar and, finally, multiphonics don't exhibit these ratios.

3.3 General Theoretical Discussions Relevant to Multiphonic Theory

Histories of the field are crucial first companions, as are syncretic studies on the state of the art. Sometimes a stubborn idea like the “subjectivity of combination tones” forces you deep into the literature to see where the vexing meme comes from. While most modern acoustics texts will mention Vincenzo Galilei (1520—1591), Baron J.B.J. Fourier (1768—1830), George Ohm (1789—1854), August Seebeck (1805—1849) and, of course, Hermann von Helmholtz (1821—1894), this won’t help you much if you find yourself looking directly at their work. For this, Hunt (1978) is invaluable. He covers the vicissitudes of acoustic theory in detail from the Ancient Greeks through the time of Newton (1642—1727). An excellent essay by Truesdell (1960) published as ancillary material in the multi-volume complete works of Leonhard Euler (1707—1783) covers the crucial intervening years until 1800. Beyer (1999) picks up the threads, bringing the conversation straight into the mid 1990s.[3] As modern as that might be, it was not until the last several years that classic debates, such as “place versus time” (i.e. frequency-detection theory versus periodicity-detection theory, regards pitch perception) or “subjective versus objective” (regards the nature of combination tones) have been largely resolved. The best syncretic study for where we stand today is *Pitch: Neural Coding and Perception* (Plack et al. 2005).

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[3] These texts are centered more squarely on acoustics. For psychoacoustic detail on Helmholtz, Carl Stumpf (1848—1936) and others, see Boring’s (1950) *A History of Experimental Psychology*. For a broader and more contemporary history (though also more cursory) and good description of the hearing mechanism see Warren (1999). The essential study for basic contemporary issues in perception and cognition is Bregman (1990).

3.4 Multiphonic Catalogues After 1960

Although there are some early outliers on our subject (for information on Bayr's early 19th century *School for Doublenotes on the Flute* see Section A.2.2), reference works on multiphonics generally began to emerge in the mid 1960s. Bruno Bartolozzi's *New Sounds for Woodwind* (1967/1982) was one of the first references of the 1960s and it had the unusual ambition of discussing each of the major orchestral woodwinds separately. Translated in English and published by University of Oxford Press, *New Sounds for Woodwind* generated an enormous surge of multiphonic activity in terms of original composition and performance and the production of further multiphonic references and theoretical speculation. However, the work did receive significant criticism for its accuracy and strategy (Haddad 2006; Zonn 1975). Heiss (1968) and Singer (1973) wrote works of similar scope but smaller size, which were nonetheless influential because of the greater accuracy of their transcriptions and the utility of their fingerings for instrumentalists in North American schools of performance.

More typically authors produced expositions for single instruments that related their own research (and this is still true today). By 1982 authoritative introductions were produced for the flute (Heiss 1966; Stokes 1970), the recorder (Vetter 1968; Margolis 1976), the oboe (Holliger 1965/1966; Zonn 1978; Roxburgh 1982), the clarinet (Smith 1972; Singer 1975; Farmer 1978), the bassoon (Lapina 1977) and the saxophone (Caravan 1974), with limited investigations beginning for the voice as well (Brooks 1974; Barnett 1977).[4] From

[4] For a comprehensive list of multiphonic resources up to 1985 see Barata (1988).

1974 full-length books and dissertations devoted to the extended techniques of single instruments began to appear. The chapters addressing multiphonics were often substantial, sometimes comprising over half the length of the total work. Between 1974 and 2009 important monographs detailing the multiphonics of single instruments, or instrumental families, were written for the flute (Howell 1974; Dick 1975; Brokaw 1979; Dick 1986; Koizumi 1996; Levine 2002; Levine 2004; Shiung 2008), the oboe (Post 1979; Veal 1994), the clarinet (Caravan 1974; Farmer 1977; Rehfeldt 1977; Haddad 2006), the bassoon (Lipp 1982; Schwartz 2006; Gallois 2009), the saxophone (Caravan 1974; Kientzy 1982; Bergeron 1989; Kientzy 1993; Gross 1998; Phelps 1998), the horn (Levesque 2005), the trumpet (Cherry 2009) and the trombone (Dempster 1979; Davidson 2005).

It is often useful to have at least two of these to cross check harmonic transcriptions of particular fingerings. Discrepancies could reflect the accuracy or aesthetics of individual authors, but they could also stem from variability between instruments, set-ups or upstream resonances. Only recently have comparative studies begun to critically assess the executions and methodologies of multiphonic catalogues created for essentially similar instruments (Haddad 2006). No study yet addresses the differing multiphonic capacities of common but dissimilar instruments, such as Oehler and Boehm clarinets, which are typically found in Germany and North America respectively. However, one study does survey physical distinctions between these clarinets, without addressing multiphonics directly (Feller 1983).

An often-discussed limitation of these studies is the presentation of individual multiphonic sonorities as conventional, if nonetheless microtonal, chords (Zonn 1975).

Singer (1975) proposed a system where different colors would help indicate timbral and amplitude characteristics of multiphonics components, but no alternative to the chordal method has emerged. Kientzy (1982) stands out for presenting several columns of additional data and commentary on each fingering he discusses. Recent books by Gallois (2009), Levine (2004) and Veal (1994) set a new standard, in at least one respect, by providing excellent and well-indexed digital recordings of all multiphonic sonorities provided in transcription. From the mid 1980s on, most works have settled on a harmonic organization wherein multiphonics are presented in an ascending order according to their lowest note, their highest note, or in one case, both (Koizumi 1996). The method of designating fingerings has still not been standardized.

Finer levels of organization are rare. Rehfeldt (1977) gives an excellent presentation of clarinet multiphonics according to chord types. Dick (1975) and Bergeron (1989) provide compelling accounts of particular relations amongst chords and fingerings but these works appear to be written primarily for performers and either cross-reference, or sometimes do not even provide (Bergeron 1989), transcriptions of sonorities under discussion.

In a humorous but telling aside, when meeting Christian Hommel, the oboist of Ensemble Modern, his first concern was *to make sure I had t*he multiphonics book of Veal (1994), his second concern was that *I already had it*. Even with the best of these books in hand, it is essential to work with instrumentalists. (For a brief background on how such relations contributed to the remarkable rise of multiphonics in the mid 1950s, see Section A.3.2)

4 Structural and Analytical Perspectives in *Radial*

Radial seeks to extend a line of sound-mass compositions that includes works by Ligeti, Xenakis, Scelsi, Lucier, Oliveros, Niblock and Murail. Although their expressive aims are rather different, the latter five composers often share a technique of composing "into" sound, which, as we noted earlier, Murail (2005b) refers to as "de-composing" in his celebrated commentary on Scelsi. Again, the basic idea of "de-composing" is to combine sounds in a way that draws attention to aspects of timbres rather than building new timbres. By creating salience at the level of the partial—usually through inharmonicity, or amplitude variance at odds with larger envelope trends—fusion is disturbed and the component elements of a sound are revealed in differing degrees. Curiously, there is a similarity of harmony and timbre between works composed in this way and the basic character of multiphonics, which come "pre-packaged" in various de-composed perceptual states. It is clear that multiphonics could be usefully orchestrated into sound-mass compositions and other styles that explore these sonorities. Murail (2007) specifically discusses them in this regard in his seminal article *The Revolution of Complex Sounds*. However, the sonorities and verticalities of multiphonics have intriguing global characteristics of their own which are little understood, and these could suggest unique linear structures and compositional logics for deploying both timbre and harmony. *Radial* pursues this line of thought.

The notion that a group of multiphonics might have particular formal implication was somewhat speculative in the early stages of composing *Radial*. The general intuition was this: conical woodwinds (see Section 2.3.1) do not have substantial sets of multiphonics that maintain identity under transposition—these are more typical of the cylindrical woodwinds, such as flutes and clarinets. Might the irregularities of conical woodwind multiphonics reveal strong and weak resonances that could be seized compositionally? Could the brittle timbral quality of certain multiphonics extend from a maximal distance between such resonances? If so, then following the timbral branching structures of Lerdahl (1987), these sounds might suggest imminent progression because at a physical level they literally have nowhere further to go. Similarly, the rich, flexible timbres of certain multiphonics might be an index to the close proximity of multiple strong resonances and the possibility of strong or weak prolongations towards those gravitational poles. The reality of saxophone multiphonics is very complex and their nature is not fully resolved here, but these rudimentary intuitions proved to be sound. A basic space that demonstrates these conditions was charted out and informs the foreground, middle ground and background of *Radial*. The first step in developing this structure was to establish the basic scalar level — a nonhierarchical "lexical" level — of alto saxophone multiphonic space. We turn to that process now.

4.1 Creating Multiphonic Space

The composition of *Radial* began with the creation of a basic space that characterizes the collection of alto saxophone multiphonics as a whole. Concepts of *Tonal Pitch Space* (Lerdahl 2001) provided the launching point for the search. Lerdahl's "basic space" is a five-tiered hierarchy with level *a* designating the octave, level *b* the fifth, level *c* the triad, level *d* the diatonic scale, and level *e* the chromatic scale. Clearly a "multiphonic space" is very different than tonal pitch space, but it is useful to discuss departures between these spaces as they arise.

A first important distinction between conventional pitch spaces and those of a multiphonic space is that multiphonic spaces are comprised of actual pitches as opposed to pitch classes. This is because identical materials are not available in each octave and the functionality of particular pitches may also differ from octave to octave. Consequently it is also true that the lowest 'lexical' level—in tonal pitch space this is the chromatic level *e* from which the hierarchical diatonic level *d* draws its basic content—does not involve pitches separated by equal intervallic values. This situation complicates conformity with both well-formedness conditions in Lerdahl's basic space (2001). Let's look momentarily at these issues more closely; there are valuable reasons, even beyond the scope of multiphonics, for considering the problems.

What is interesting, and of more general value than the topic of multiphonics themselves, is that these issues lead to subtle considerations of top-down and bottom-up influence in pitch spaces. Tonal pitch space puts cognitive force and hierarchical influence at the top of the space. These push down the structure to pick more passive materials from lower levels as needed. Ultimately, the passivity of lower levels depends on the chromatic sameness—the flatness—of the lowest level, which is level *e*. But what happens in a well-tempered or meantone system, where adjacent interval classes are somewhat different, and can project undesirable roughness into particular selections? Clearly there is a residual resistance that makes some modulations by fifth more distant, and in certain cases even prohibitively distant. (This issue is discussed further in Section A.2.3.)

A related problem occurs in instruments with an uneven availability of pitches in different octaves. Imagine a strange instrument microtonal above C5, chromatic down to C3 and diatonic in C Major below C3. Top-down influence still has its force, and if the instrument appears in one or two pieces we may never know its limitations, but eventually we will know them and this could effect our perception of pitch space paths in that context. For instance, does a harmony tense or relax in a downward arpeggio if we know that it can't be prolonged after two more consecutive pitches? It is as though the very timbre—or sonic presence—of the instrument comes to signify possibilities and prohibitions. There are many better-known and very real cases. Lerdahl (1987) discusses this situation in regards to Classical period brass and timpani. Section A.2.3 obliquely touches on this issue through discussion of the different flutes needed for Mozart's writing versus Haydn's writing. Our imaginary instrument would also support many well-known relaxing gestures from spectral composition. (Absent such hierarchical material, Ligeti often seized upon the limits of register itself in an almost comic manner to generate tensing and relaxing cadential functions.)

Considering these cases, the sense in which our multiphonic issues are "misdemeanors" becomes more clear. A system that repeats every minor 9th violates octave equivalence in a different way than a system that lacks same content in each octave but still admits identity at octaves. Similarly, spaces that have distorted interval sizes in particular regions, or have small microtonal divisions in limited areas of the total chromatic, don't necessarily force a radical change in the perception of step versus skip or other broader intervallic identities, but they do contribute to the value or meaning of harmonic and melodic movement in those particular areas. The question is at what point is it valuable to draw a pitch space with a variegated or tessellated surface that explicitly notes "missing options" or "warped distances" over which the pitch space paths of a piece are drawn? Analytically, for instance, it may be to the point to draw pitch space paths for a Classical period horn concerto on a scaled down regional map. Surely in conjectural or exploratory spaces, it is useful to draw in as many "tautological" restraints as possible. In this spirit we entertain the creation of a rather unusual alto saxophone multiphonic space, with somewhat different meanings for how we "hear down into it" and what resistances it "gives back."

4.1.1 The scalar levels.

Multiphonics are generally thought of as chords (or timbres) in particular voicings. These cannot be abstracted and realized in different octaves and different inversions. Therefore they are not made of pitch classes, but that doesn't mean they are not, at times, also pitches that can be played singly and successively. On the saxophone and other woodwinds, multiphonics are ultimately fingerings, and the sonorities they produce are often able to transition smoothly from many tones down to just three, two or even one note (Coltrane's subtle use of this distinction between note and chord is discussed in Section A.3.1). The first questions then to ask, are "which pitches are actually used?" and "what is the length of this space?" A multi-octave scalar level is pertinent to any multiphonic system. To settle this we establish a unique level *e*.

The smallest step in alto saxophone multiphonic space is substantially smaller than the semitone; it could be considered an eighth-tone or less. Functionally, the lowest scalar level is best described at two heights of resolution: the quartertone level (*e2*), which more readily reveals collective trends, and the eighth-tone level (*e1*), which shows critical harmonic and melodic resolution, but may at times go beyond a practical margin of error.[1]

Figure 4.1 shows all pitches used in the alto saxophone multiphonics ordered in an ascending scale. The data is realized at eighth-tone resolution (level *e1*), but double barlines specify larger quartertone groupings (level *e2*). F quarter-flat 3 is the lowest pitch in the collection and Db6—three octaves above the fundamental—is the highest pitch in the collection. This means it is the highest component of a multiphonic that can be sustained leading into or coming out of a particular sonority. Higher tones can be discerned in alto saxophone multiphonics, but they neither behave, nor or heard, with the same degree of independence. They are perhaps analogous to the "fused" harmonics of conventional

[1] Specifically, variations encountered within a particular multiphonic, when moving from instrument to instrument, or player to player, may be greater than this fine level of characterization. However, contemporary research (Kientzy 1982; Levine 2002, 2004; Shiung 2008; Veal 1994) usually emphasizes that mere discrimination at the level of the quartertone lacks descriptive power and that multiphonics do have significant stability from performance to performance and performer to performer. Amongst modern critical works Gallois (2009) alone finds the quartertone level to be sufficient for his transcriptions. However his general approach is quite different. He only gives 60 harmonically distinctive chords, which are very rich, ranging from four to six notes. Other authors often give accurate accounts of between two and four hundred multiphonics, many of which are transparent two and three note sonorities in finely tuned variations.

pitches, which with focused concentration can be sorted out by different listeners to varying degrees. For present purposes, the upper limit is given here as Db6.

Given these upper and lower limits, multiphonic complexes nearly cover the entire range of the instrument. F3 is a major third higher than the alto saxophone fundamental of Db3 and a virtuoso performer can play to around Ab6, a perfect 5th higher than Db6. However, multiphonics have a more homogenous registral effect because they appear in chordal complexes that usually span one to one and a half octaves. Db6 may be the highest soprano position tone and F3 the lowest bass position tone, but G4 is the *highest bass position tone*—a mere major 9th above F3.

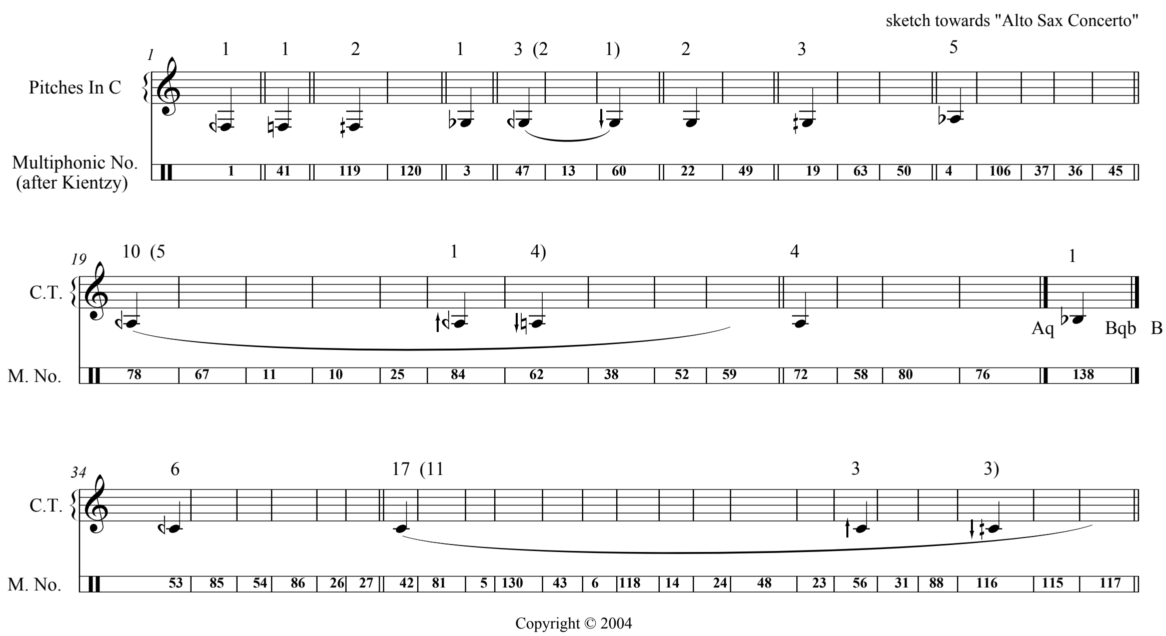


Figure 4.1: All pitches used in alto saxophone multiphonics ordered in an ascending scale. Double barlines emphasize quartertone groupings. Numbers in the lower staff indicate which multiphonics, following Kientzy's (1982) ordering, incorporate the pitch. Numbers above the staff summarize each note's "resonance depth" (i.e. how many multiphonics use that particular pitch.)

Figure 4.1 (Continued)

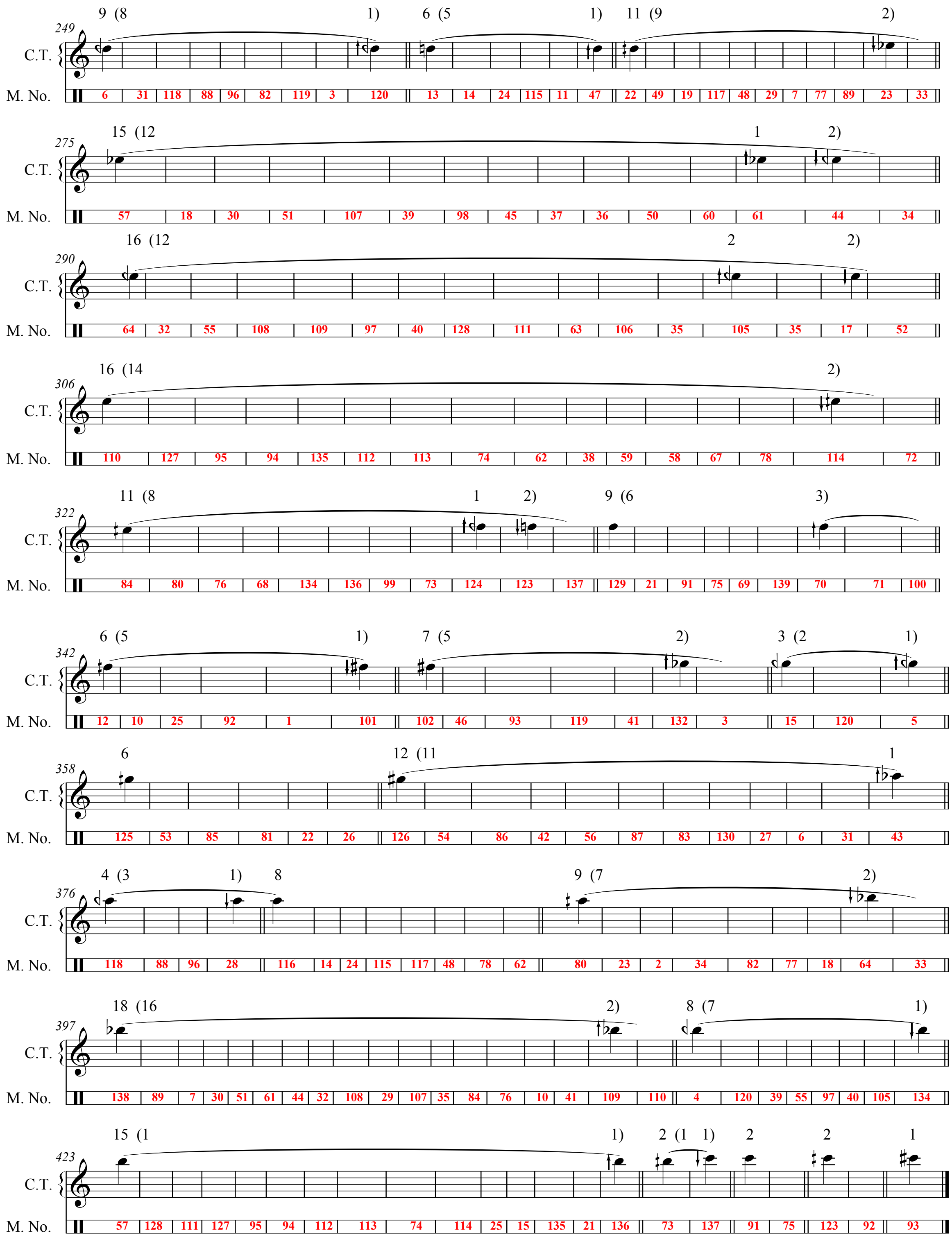


Figure 4.1 (Continued)

Intriguing trends emerge from Figure 4.1. Eighth-tone scale steps are common. But they are a significant minority in the bottom octave of the saxophone, where they divide only three of the 17 quartertones present. By contrast after Db4—which is the second harmonic of the fundamental, though rarely performed that way — a total of 34 out of 48 quartertones (71%) are divided into smaller eighth-tone intervals. This asymmetry is deeper than the issue of microtonal fingerings. Most woodwinds have a greater number of microtonal fingerings above the first octave, but we might have guessed that a variety of difference tones could have emerged in the area of the lower octave as a result of higher pitches in multiphonic sounds. Presumably this would have created at least a few unusually tuned notes. But this is not the case. Such difference tones—those appearing in the bottom fifth of a multiphonic—only begin to arise from sonorities with bass position tones at C4 (one half-step below the second harmonic of the instrument's fundamental) and above. Chord "types" or driving tones that could produce such intervals are common in the lower register, but they simply don't yield these difference tones, regardless of the dynamic at which they are performed (see Figures 4.2 and 4.3 below).

Figure 4.1 also shows a notable group of quartertones and semitones that do not appear in any multiphonic at all. Measures 32-34 and 227-229 show that A quarter-sharp 3, B quarter-flat 3, B3, B quarter-flat 4, B quarter-sharp 4 and G5 are all missing from the set of pitches found in alto saxophone multiphonics. Moreover there are several multiphonic chords (see m. 138-139, Figure 4.12) which are quite difficult to explain unless we imagine these "missing" notes are present, but somehow filtered out. Therefore it seems that,

Figure 4.2: Eighteen multiphonics below C4 that could but do not create low difference tones. These chords have the arithmetic relations amongst driving tones needed to create differences tones in the bottom fifth of the chord, but no such tone is produced. For reference, the bottom staff shows the sonorities transposed to bass position C4.

beyond being coincidentally absent, these tones resist appearing even when they are called for. It is as though they are dead spots, or anti-resonances.

After G4 (m. 187) all additional pitches listed in Figure 4.1 are found in the middle or upper voices of the set of all multiphonics. No multiphonic bass positions exist above this point. It is noteworthy that some middle and upper position pitches such as E quarter-flat 5, E5 and Bb5, belong to as many as 16 to 18 multiphonics and often have a variety of tones resting underneath them. These are powerful resonances, both in terms of "resonance depth" and harmonic reach; they are the opposite of anti-resonances.

Figure 4.3: Eighteen multiphonics between C4 and D#4 that create low difference tones. These chords have the arithmetic relations amongst driving tones needed to create differences tones in the bottom fifth of the chord. They do so, but not in an easily predictable manner. (This latter issue is discussed in detail below.) For reference, the bottom staff shows the chords transposed to the common bass note C4.

Scalar levels *e1* and *e2* — two-octaves plus one quarter-sharp minor sixth in length — demonstrate the weaknesses of multiphonic listings organized by bass position alone. Vital trends are also evident in upper position pitches. Every appearance of a pitch must be noted to capture its depth and the variety of structural roles it plays. Finally, bass position tones are not strictly roots and can at times have minor aural salience. (For one representative case, see Multiphonic No. 74, in m. 35, Figure 4.3.)

One could object, however, that appearances of particular notes, especially in middle voices, are often mere difference tones, the simple by-products of other tones, and that it is loquacious to list them all. This notion is quite mistaken. Section 3.2 (and appendix B) emphasized that multiphonic descriptions conceived by, or reduced to, simple mathematics have weak explanatory power. For instance, Figure 4.2 showed that combination tones are not always produced by similar structures; and Figure 4.3 showed that those which do appear, often do not manifest in predictable locations: different chords in m. 20 and 34 improbably produce the same difference tone; the opposite is observed in m. 29 and 30 where same vertical sonorities produce separate interior difference tones. Another layer of distortion must influence the precise positioning of these tones.

As will be seen (Figure 4.12), all but two difference tones that appear in the lower fifth of a multiphonic also appear in other multiphonics as "principal" collateral tones—which can be played separately from other notes in the sonority. Therefore, if a tone doesn't have a life as a driving tone (collateral tone), it likely doesn't have a life as a difference tone either. This point is critical. One could object to the general idea of "powerful resonances," claiming instead that resonance depth is a simple artifact of the permutational variety of fingering systems. To be sure, that is one factor (discussed in Section 2.3.1). However, multiphonics are nonlinear sounds, and this nonlinearity principal extends from three variables, one of which is friction. (See Section B.2 for more on the physical conditions underlying non-linearity.) A central source of friction in woodwind instruments is the disruption of the bore by keyholes and this disruption is present in the bore whether or not the keyhole is opened or closed. Therefore all fingerings *modify these disruptions*; it is not the other way around. Thus one problem of chiefly relating multiphonics to fingering patterns is that harmonic relations between “unrelated” fingerings may exist on a deeper physical layer than the fingerings themselves. In sum, a revealing multiphonic description should seek to find powerful resonances and voids across the instrumental range, and then observe how chords, difference tones, and fingerings interact with these. Such an initial holistic description is given in Figure 4.4.

Figure 4.4 the “Preliminary Quantitative Resonance Curve for the Alto Saxophone Multiphonics” first reveals powerful resonances at C4, D quarter-sharp 4, E quarter-flat 5, E5 and Bb5. It also shows three valley regions around B3, B4 and G5, which reach to the staff marked “0” indicating that these pitches are our resonance voids discussed above that appear nowhere in the multiphonic set. The space is anything but flat. Most importantly these large variations happen in clear trends. For instance, the single multiphonic including Bb3 is surrounded by two resonance voids. The area is a *collective indication of weakness*. Although the resonance void at G5 is solitary, it is precipitated by a noticeable drop in the resonance depth of several quartertones beforehand and one quartertone afterwards. Large peaks are also the products of several terms. It is notable that sampling at the eighth tone is significantly more jagged. Such differences may be important locally, as surface variations, but attention to the whole at the level of the eighth tone obscures the global outlook.

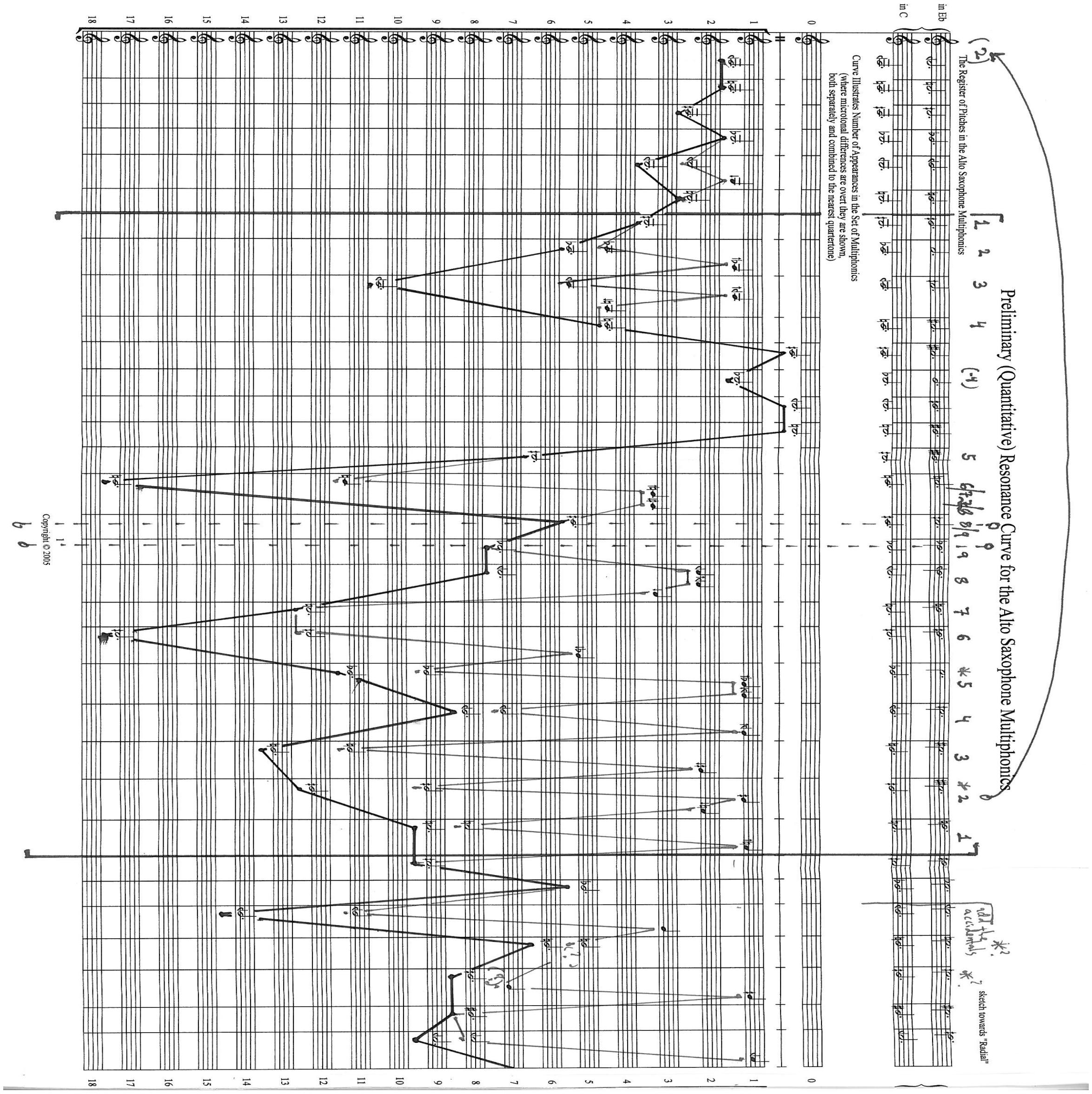


Figure 4.4: Preliminary quantitative resonance curve for the alto saxophone multiphonics. The darker outer line shows data at the quartertone level. The smaller interior line shows data at the eighth-tone level. Numbers to the left and right of the system state how many multiphonics incorporate each note (i.e. "resonance depth.") The bracketed area shows the range of bass position tones explored in *Radial*.

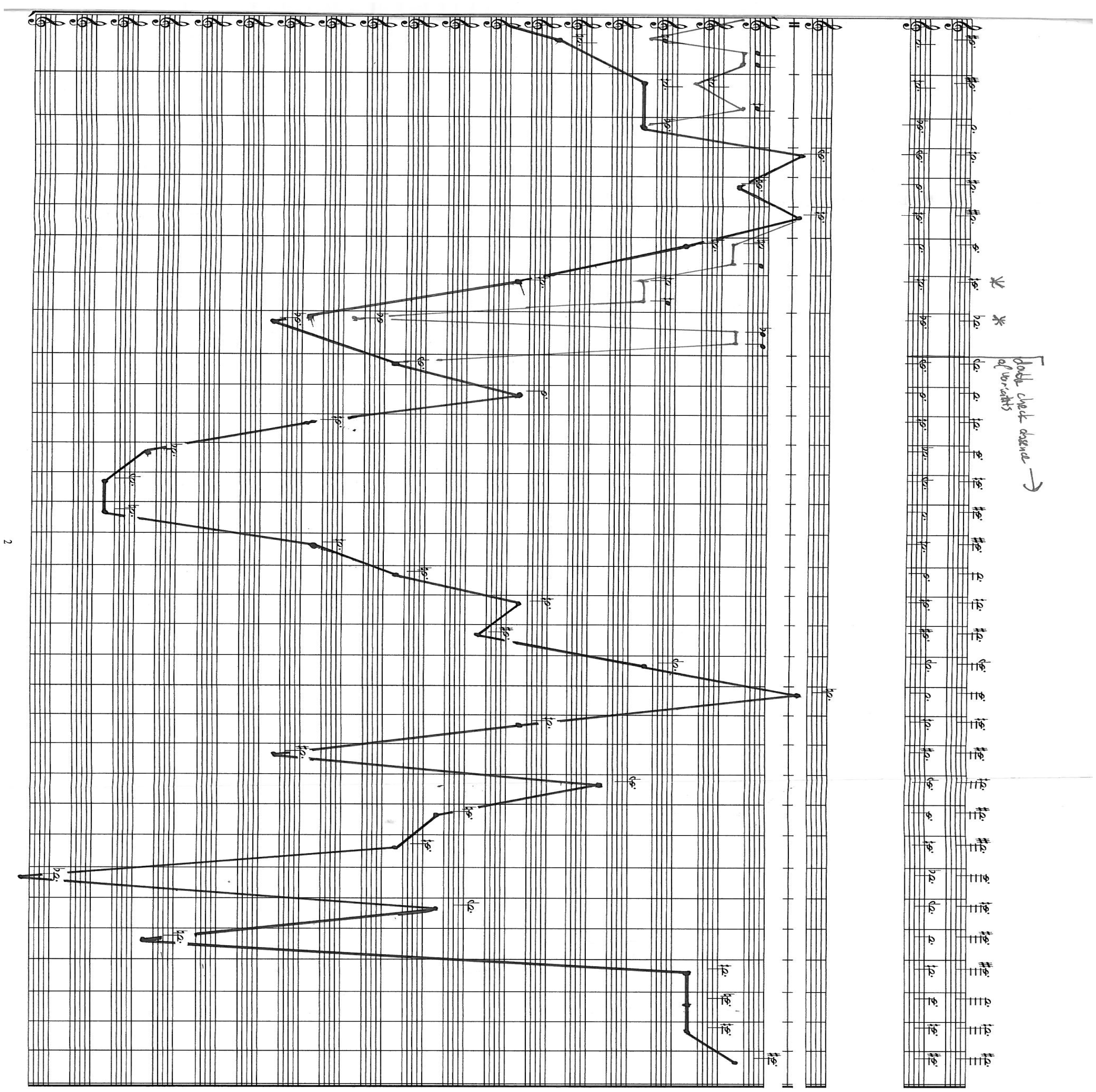


Figure 4.4 (Continued)

Before moving on, it is valuable to ask if there is a meaningful level *d*—a hierarchical scalar level—in alto saxophone multiphonic space. Ultimately such levels come from hearing and composing works. However, the system is constrained enough that it does seem to have limitations at least on par with the "key colors" of tuning systems discussed earlier. For instance, composing a work around "B" would be deeply prohibitive in this system considering that it and its neighboring tones fail to appear in two-out-of-three octaves. By contrast Ab/A seems quite promising. Thus, at the very least, level *d* could be outlined as a compositional tool to orient projects towards areas "of material." That said, adjacencies around voids are typically interesting if not rich in membership, one could theoretically take these few cases as a starting point leaving the girth of the total multiphonic body aside. Nonetheless, few composers would wish to remain ignorant of the following level *d* and the dialectical case of the "anti-level *d"* just discussed.

Figure 4.5 shows levels *e2* and *e1* together, uncluttered by the number names for specific multiphonics. (Figure 4.1 thus remains a key reference.) Instead, it shows the resonance depth for each quartertone, with brackets breaking those numbers down to the eighth-tone level. Level *d* is provided at two resolutions. The first, "level *d1*", allows peaks with resonance depth of six or more to rise to its hierarchical level. "Level *d2*" is more selective only allowing peaks with ten or more members to rise to its hierarchical level.

The results are striking: the eighth tone level drops out completely with a near complete quartertone scale arising from C quarter-flat 4 to B5. Gaps at A3, F#4, A4 seem somewhat artificial—lower, though not insubstantial, peaks dropped from level e can readily fill them in. However, gaps at B3 and B4 force considerations about where leading tone harmonies can be created. They carve out modal asymmetry much the same way Scriabin's mysterium set forcibly adds it in. (That scale, of course, can be freely transposed, whereas these voids are firmly etched in place.)

Level *d2* is more surprising. Large gaps force confrontation with the clear pentatonic outline “A3—C4—D4—E4—G4”. A larger interval skips A4 but the sequence picks up again, one semitone higher, with “Db5—Eb5—F5—Ab5—Bb5”. This reduction leaves out some

Figure 4.5: The scalar levels of alto saxophone multiphonic space. Level *e* shows all the tones present in the set of multiphonics. Level *d1* and *d2* show scalar selections that privilege consecutively larger resonance depths.

Figure 4.5 (Continued)

chromatic and quartertone inflection, but the outlines are plain enough. Observing "pentatonic" level *d2* of multiphonic space it is impossible not to mention the simple folk and blues melodies Albert Ayler (1965) choose as launching points for his unparalleled multiphonic improvisations. Analysis is needed to verify precise correspondences, but from here this striking (though at the time often deeply criticized) confluence in his music seems inevitable—and ready to be born again.

Ultimately level *d2* is a particularly powerful transposition of a pentatonic state, where each scale member has homogenous resonance depth, meaning a comparable range of resources is available, if not precisely the same materials. That uniformity can be explored through pentatonic modal transposition at level *d2*, but cannot be uniformly transposed to other keys. For instance, transposing the unit one half-step to Db4 produces the scale segment "Bb3—Db4—Eb4—F4—Ab4", but particular scale members at this transposition (especially Bb3 and Db4) have limited and sometimes very unusual tone colors. Thus, transpositions of level *d2*—allowed by level *e*—are possible, but would exhibit differing degrees of timbral distance.

Regardless of the scalar level one chooses to approach alto saxophone multiphonic space, it does have asymmetrical qualities that give meaning to a limited set of harmonic modulations, all modal transpositions and local sequential treatments of material. This is not a flat space built upon the timbral and/or microtonal equivalent of a chromatic scale. As will be shown below, *Radial* takes its launching point from the asymmetries of resonance depth and intervallic step size and timbral color that arise between ascending and descending materials mutually honing in on the center point of Db4, one octave above the instrument's fundamental tone.

But for now, having established basic issues regarding the scalar levels of multiphonic space, it is time to ask what verticalities extend from or enmesh these individual tones. After all multiphonic space is surely a chordal space.

4.1.2 The prototype multiphonic sonority.

We now turn to Lerdahl's discussion of timbral hierarchies (1987) to ask if there is a prototype sonority that appears at various transpositions in level *c* (the standard chordal level of pitch space), which also undergoes distortion in various timbral dimensions. Lerdahl suggests that timbral prototypes arise from stability conditions, pertaining to a style and to psychological factors that designate particular states of a timbral continuum—such as brightness, vibrato, envelope, roughness and inharmonicity—as intrinsically more consonant, central or stable than others. Timbral prolongations arise from movements, in any one of these dimensions, to and from the prototypical state. Because timbre is multidimensional—involving any of these and other factors—a particular sonority could be prototypical in one sense, while being relatively unstable in another. Multiphonics exhibit such behavior.

Perhaps surprisingly, most of the alto saxophone multiphonics—116 out of 140 identified by Kientzy (1982)—can be related to a central figure, which is most readily described as a semi-dilated version of the first three partials of the harmonics series. It undergoes harmonic distortion of compression in one direction and further harmonic distortion of dilation in the other. Figure 4.6 shows the prototype transposed to bass position C4. The tenor position element is a minor 9th above the bass position (of course stretched relative to the harmonic series.) The alto position element—which is often the highest member of the sonority—is consonant relative to the tenor position, but stretched relative to the bass position.

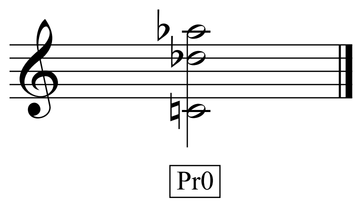


Figure 4.6: The prototype alto saxophone multiphonic (Pr0). Seven multiphonics have this precise structure but more than 80 multiphonics are similar enough to be considered stable variants of this identity.

Section 2.3.2 explains the physical basis of perceiving a stretched harmonic series-structure as "prototypical." It is the default condition of most harmonic series-based multiphonics on the saxophone and therefore generally sounds strong. It is also in the middle of the set, and significant dilation or compression away from that point brings clear "steps" of distortion and instability beyond the measure of inharmonicity itself. For instance, even multiphonics with bass-tenor octaves are more notably tense and unstable, have other distorted partials, and are insignificant in number. Figure 4.6 is the best description of the "center" for more than 80 multiphonics; about 30 more also belong to this general identity, but their greater distance in the inharmonicity continuum makes them considerably more idiosyncratic. Figures 4.7 - 4.9 show the process of stability branching from the central prototype and detail places where "steps" of inharmonicity create significant reorientations of identity. For clarity all cases are shown transposed to C4. However, as mentioned earlier, actual bass positions are found at most quartertones from F3 to G4.

Figure 4.7 describes 30 multiphonics with the prototypical bass-tenor position. Of these only seven have the central alto position Ab5. The remaining 23 alto positions are slightly more dilated or more compressed, but individual instances do not go beyond the limits of “high” A quarter-flat 5 and “high” G quarter-flat 5 shown in Figure 4.7.

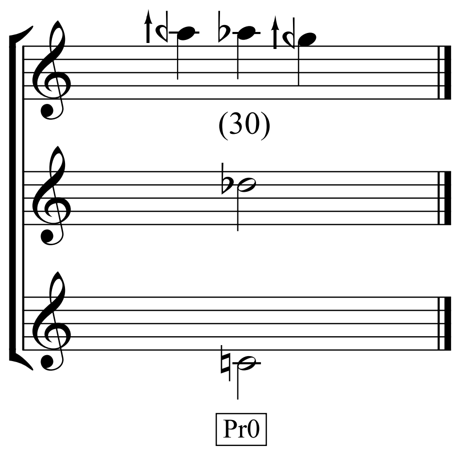


Figure 4.7: The prototype multiphonic (Pr0) in detail. Thirty multiphonics have this structure. Seven incorporate the central Ab5 alto position (which is given alone in Figure 4.6.) The other 23 alto positions do not go beyond the upper and lower limits shown at top right and left.

Dilating the bass-tenor interval by one quartertone is audible but does not disturb the essential strength and character of the sound. The same is true of compressing the bass-tenor interval by one quartertone. Considering their aural quality, multiphonics with those intervals are best labeled and treated as stable, but particular, variants of the prototype.

Figure 4.8 adds these chords to the prototype collection. Dotted barlines (and group names) indicate their place in the wider "central" region of the prototype. The dilated tenor-bass prototype (PrDi0) appears in 24 multiphonic sounds. That larger interval does not lead to higher alto positions, though it does seem to prohibit bass-alto intervals smaller than "C4 to G quarter sharp 5". The compressed tenor-bass prototype (PrCm0) is more odd. In spite of the bottom compression, its alto position tones have *a higher "low limit"* (G quarter-sharp 5) than those of Pr0; its upper limit does drop by about a quartertone relative to alto positions of the other cases.

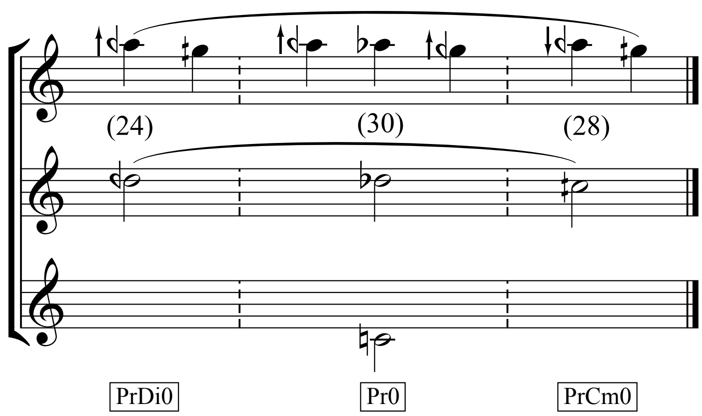


Figure 4.8: The wider central region of the prototype multiphonic (PrDi0—Pr0—PrCm0). 82 multiphonics share this basic harmonic character and support similar ranges of stability.

Further levels of dilation and compression create significant changes of character, indicated here as "numeric steps" away from the wider central region. The prototype supports six further steps of compression and four further steps of dilation before fundamentally changing or loosing identity.

The few examples *wider* than these limits are best thought of as multiphonics with missing tones; they appear to interact with resonance voids and it therefore oversimplifies matters to call them instances of dilation. Even PrDi4 has a separate physical basis than other members of the set, but is kept here for its aural proximity to liminal cases. Multiphonics smaller than these limits are not artifacts of compression. It is a mistake to relate them to progressive collateral disturbances of harmonic series-based multiphonics. All such examples pertain to a unique register that produces close collateral neighbors, which also manifest as low position difference tones. This region will be discussed shortly.

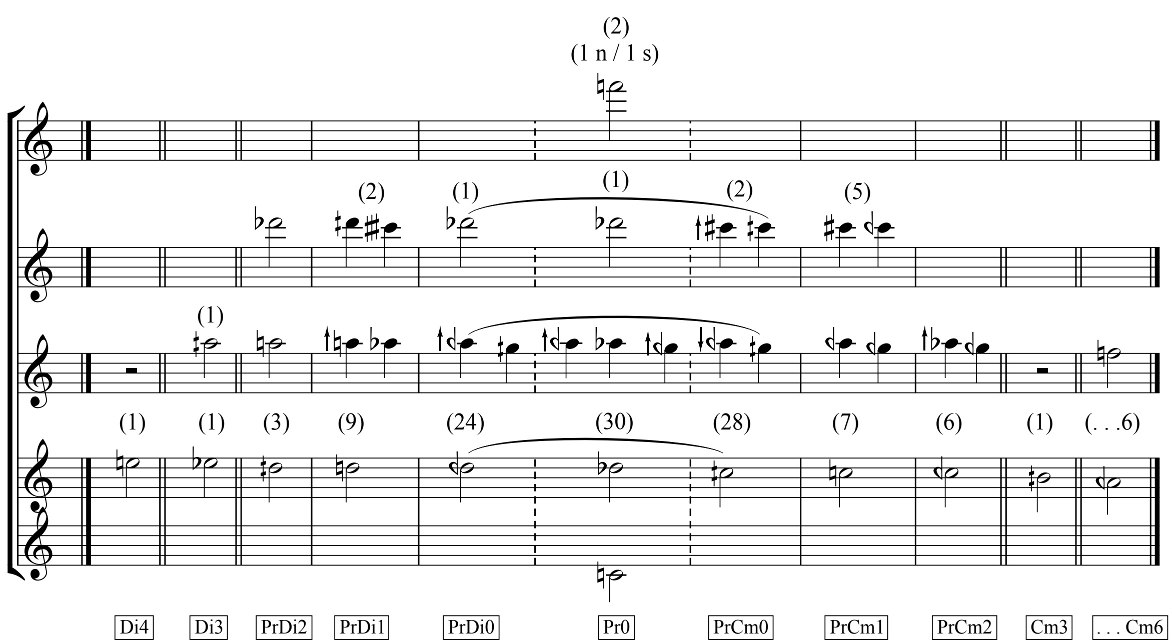


Figure 4.9: The full inharmonicity continuum of the prototype multiphonic (PrDi4—Pr0—PrCm6). 116 multiphonics (out of a total of 140) share a basic harmonic character and support similar ranges of stability.

Figure 4.9 shows the full range of inharmonicity supported by the prototype multiphonic. Double barlines separating “Di3 and PrDi2” and “PrCm2 and Cm3” suggest the basic character of the sonority has altered substantially. Depending on context, these multiphonics might be taken as associated with but beyond the prototype. Figure 4.9 further shows that some sonorities have soprano position tones and that two have double soprano position tones. One case is a normal multiphonic with multiple collateral tones; the other is an unstable “subtone” multiphonic. A curiosity revealed by this chart is that alto and soprano positions clearly resist dilation and compression comparable to the range of motion between tenor and bass positions. The manner in which these voices are “stuck” while greater compression and dilation happens below them violates the basic character of combination tone relations. Were these simple combination tones, we would observe clear contrary or corollary sweeping of adjacent members, depending on whether motions are derived from cubic or quartal non-linearity (Hindemith 1937/1945; Winckel 1960).

4.1.3 An independent collateral resonance zone.

Mention has been made of multiphonics with a total intervallic space of less than a fifth (i.e. smaller than the bass-tenor interval alone of [PrCm6]) and of typical prototype multiphonics (about a major 9th plus perfect 5th) with low difference tones in the bottom fifth of the sonority. Both these cases are related. There is a unique register in the alto saxophone multiphonics from C quarter-sharp 4 to A4 that produces a limited group of “small dyad” multiphonics, such No. 16 as “Eb4 and F4” (see Figure 4.10). If the bottom fifth of a prototype multiphonic falls within this region, one—or in some cases two—of these low tones may be added to the bass region of the sonority. These notes can appear as difference tones or as full-bodied collateral tones that can be separated from the total sonority. Such altered multiphonic chords do not transpose out of this register (see Figure 4.10).

There are, additionally, three multiphonics with similar effects (the production of small dyads or bifurcated tones) that occur in relation to *tenor position tones*. One case (No. 120) is low enough that the tenor bifurcation (or difference tone) is G quarter-sharp 4, which falls within the range identified above. The next case (No. 78) produces a tenor bifurcation at B4; the final case (No. 134) produces a tenor bifurcation at Db5 (see Figure 4.10). If these examples are added to the bass position cases discussed above, the total register for the independent collateral resonance zone becomes one octave plus one quartertone, stretching from C quarter-sharp 4 to Db5.

Figure 4.10 shows various types of sonorities produced in this region. An example with the lowest bifurcation tone (given enharmonically as D three-quarters flat 4) is seen in m.2; the highest (Db5) is shown in m. 14. All three tenor-position bifurcations are shown in m. 12-14. “Small dyad” multiphonics, where each note can be played separately from the other, are shown in m. 10-11. Prototype multiphonics with added low thirds—one open, the other as the outer limits of a cluster—are shown in m. 7-8.

Of the many prototype multiphonics that interact with this region, 18 produce low bifurcations near enough to the bass position that they sound like ‘alternate bass positions”

Figure 4.10: Various sonorities from the independent collateral resonance zone. The first three have high harmonicity and low roughness resulting from the nearness of the double bass positions. Measures 3 to 6 all have the difference tone Db4. Measures 7 to 11 have larger low dyads, both in prototypes and alone. The last examples are all three tenor position bifurcations.

creating an ambiguity about the overall ambitus of the sonority. Of these, three are the most striking. They have low roughness—resulting from “alternate bass positions” a mere quartertone apart (7 Hz)—and high harmonicity (see m. 1-3, Figure 4.11). Tellingly, Section 2.3.1 noted the flute also has a set of non-transposable multiphonics, which arise from a register of unique pitches beginning one octave above its fundamental. It is reasonable to assume that such a register can be identified for the other woodwinds as well.

4.1.4 Subtones and the instability continuum.

The multiphonic prototype and its extensions are arranged in the continuum of inharmonicity. However, a distinct "stability measure" related to roughness, flexibility and rigidity also affects them. This will be called the beating/instability continuum. It acts separately, or in addition to, any distortion or roughness arising from inharmonicity. At a basic level it can be measured by how many notes may be separated from and played independently of the multiphonic sonority. The most stable prototypes have at least three tones that can be separated from the basic sonority and this lends them a measure of flexibility perceived as girth and stability. Prototypes with only two sounds that can be separated from the sonority are comparatively more unstable or with "interior stress."

The most unstable multiphonics, in this sense, are those with only one tone (or none) that can lead into or from the multiphonic. These are the most brittle or restrained sounds in the instability continuum and their harmonies are the most eccentric. They include low dyads, prototypes with low dyads, prototypes with four voices, steeply branched prototypes, and unique verticalities. None are conventional three-voiced prototypes from the wider central region (PrDi0—Pr0—PrCm0). It is convenient to label them "subtones" (produced top-down), but several cases extend from other positions. For instance No. 121 begins from bass position, and No. 22 begins from alto position. The full group is given in Figure 4.11.

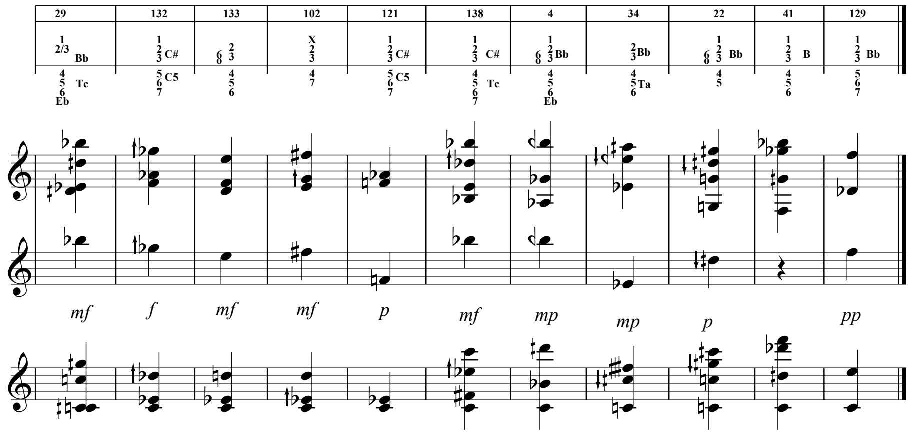


Figure 4.11: The "subtone" multiphonics. These are the most tense sonorities in the instability continuum. Only one note (or none the case of No. 41) leads in or out from the sounds. Kientzy (1982) identified these eleven instances.

If a multiphonic can be generated by only one single tone, then that tone is invariably a high scoring tone in the Quantitative Resonance Curve (Figure 4.4), contributing to peaks of resonance depth seven or more. A weak scoring tone is never the *sole generating element* of a saxophone multiphonic. Multiphonics can be played on weak tones, but they always break into sounds with other more stable generating elements. Conversely, it is the stable tones that stretch out to the weakest elements. For instance, Bb3 and B4 both have resonance depth one, and thus appear in only one multiphonic. However, the single Bb3 occurs in multiphonic No. 138, a "subtone" generated by Bb5 (depth 18) that also contains resonance at E3 (depth 13). In a related case, the single B4 occurs in multiphonic No. 78, which is generated by three pitches (the most stable variety), and those pitches are A quarter-flat 3 (depth 10), E4 (depth 16) and A5 (depth 8). The larger point here is that powerful resonance peaks do not only create stable sounds. Their power also seems important in allowing them to support unstable elements and generate unusual sonorities that don't have the force to manifest elsewhere. This means the strongest timbral progressions (movement to unusual, unstable sounds) always allow for strong or weak harmonic prolongations (via common tones) into more stable groups of sonorities. If an organizing system strictly emphasizes timbral variety or timbral stability, such harmonic links may easily be lost. Therefore the Quantitative Resonance Curve, a kind of raised relief level *e*, seems to be the most useful reference for any more particular multiphonic set under discussion—it is a powerful control for more selective organizational schemes. Given any sonority under consideration, it quickly relays broad data about the life of each particular tone in the complex. In short, were a fuller pitch space model for multiphonic space to be developed, then the Quantitative Resonance Curve would represent each pitch's potential to be developed as a common tone. (In TPS this would be each pitch's *k* variable "potential.")

4.1.5 Reflections on the basic space.

Having identified 1) the prototype multiphonic and its wider membership through the inharmonicity continuum; 2) the idiosyncratic collateral resonance zone, together with the unique dyads it creates and its interactions with prototypes; and 3) the "subtone" multiphonic which generates the most tense members of the instability continuum, some of which are totally unique to that set, we have accounted for all 140 multiphonics identified by Kientzy (1982). These unifying descriptions provide powerful intuitions about a body of multiple sounds that is dizzying when taken individually. Figure 4.12 stresses this by giving all multiphonic sonorities transposed to C4, organized by expanding bottom interval, with same bottom intervals organized by expanding upper intervals. Thus multiphonics interacting with the collateral resonance zone are shown first, followed by the continuum of prototypes (that are unaffected by the collateral resonance zone.) “Subtones” are mixed in by interval. Even in this highly ordered state, the wide and unusual range of sounds embodied in these categories is plainly evident.

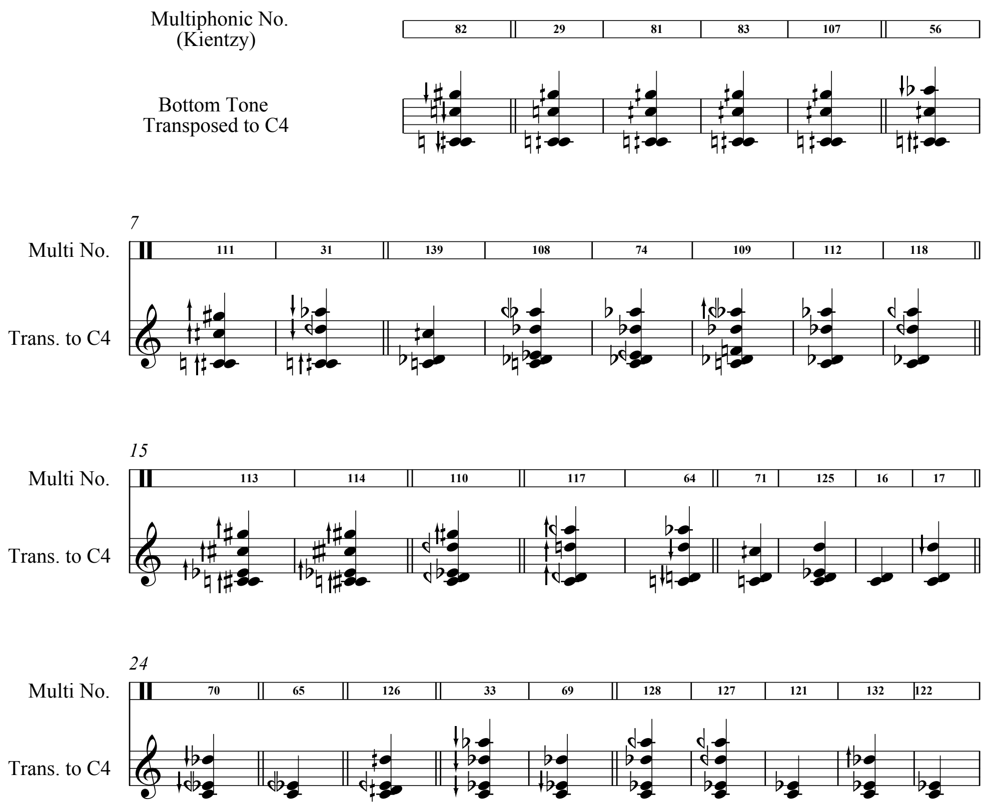


Figure 4.12: The alto saxophone multiphonics transposed to C4. All multi-phonics are organized by expanding bottom interval. Like bottom intervals are organized by expanding upper intervals.

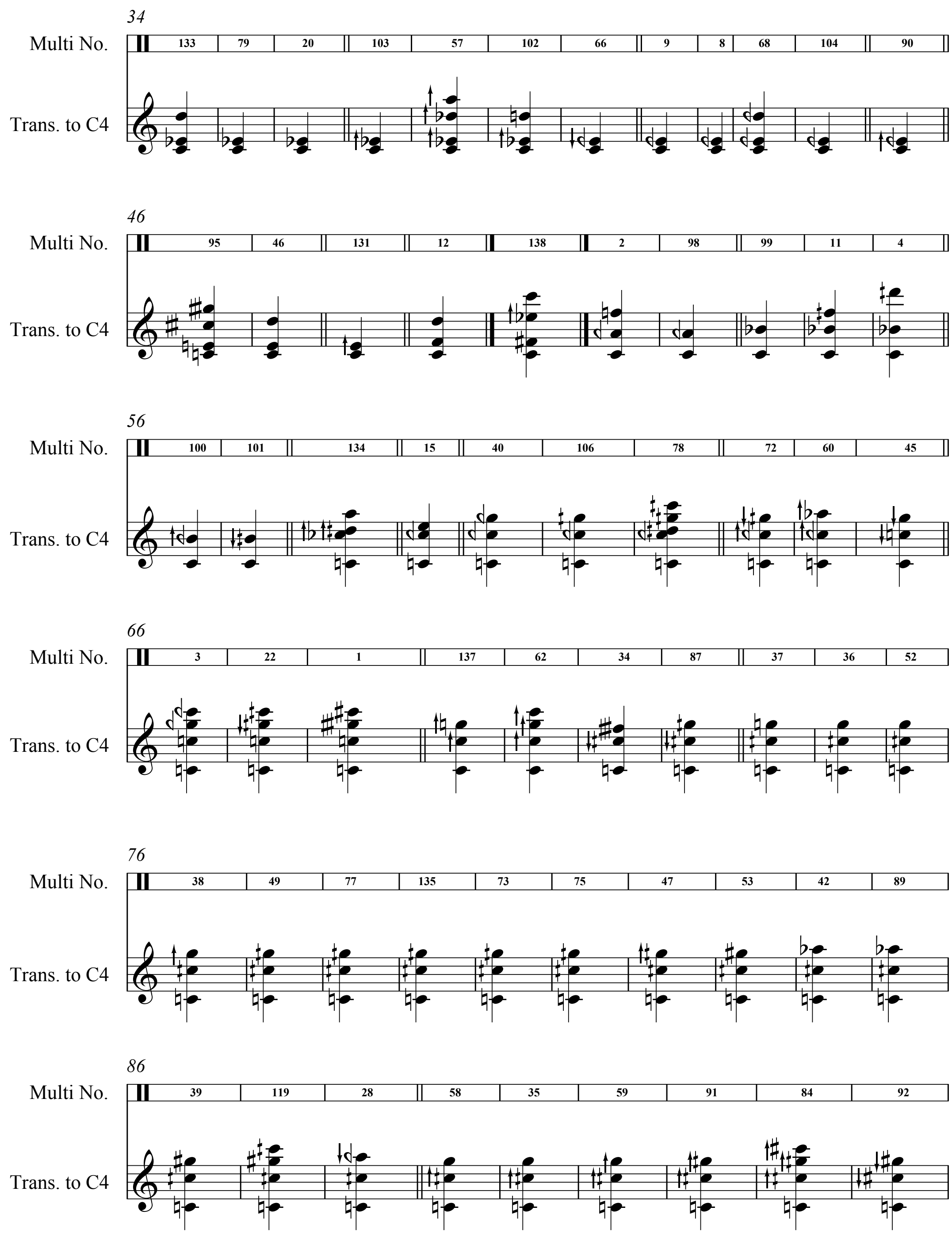


Figure 4.12 (Continued)

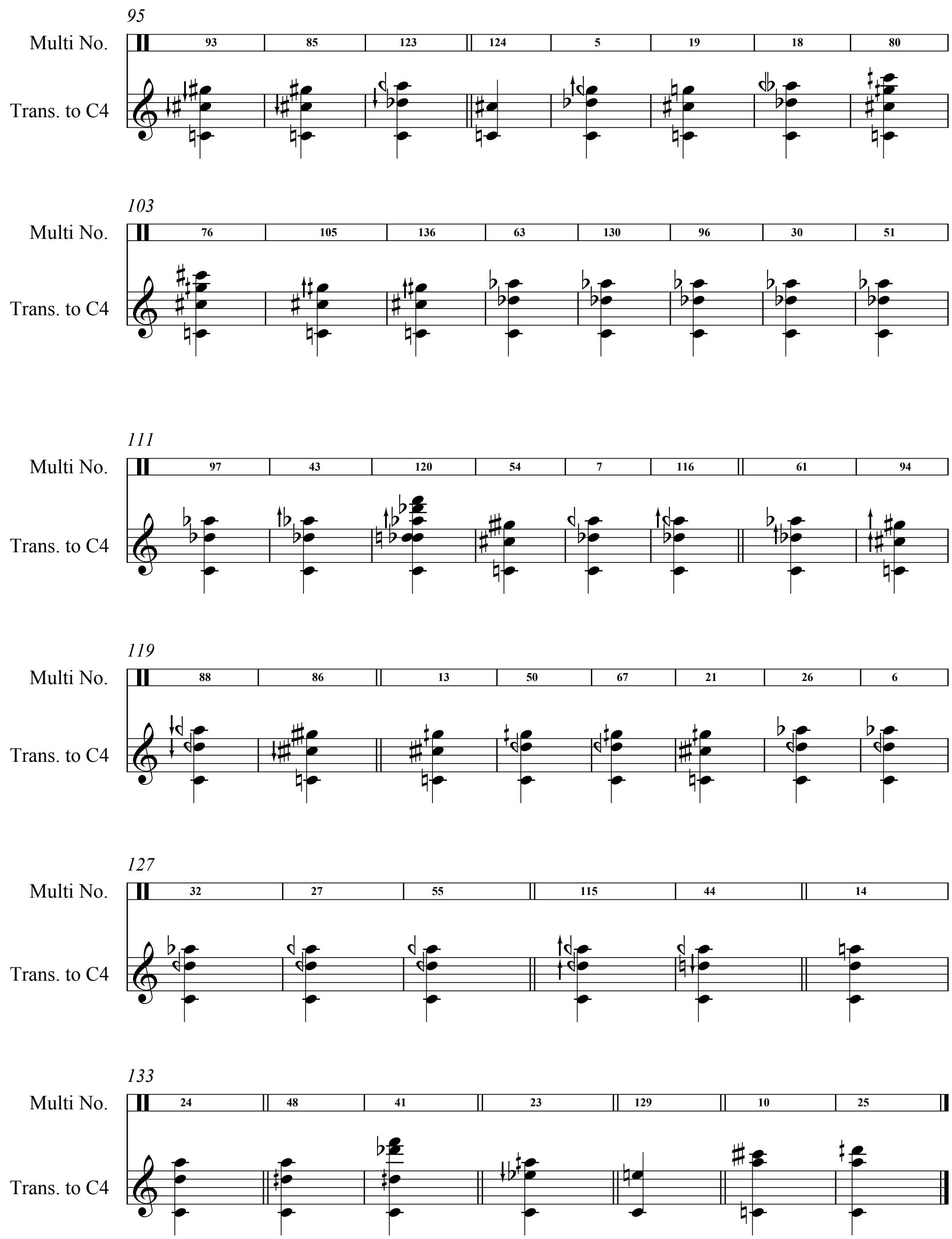


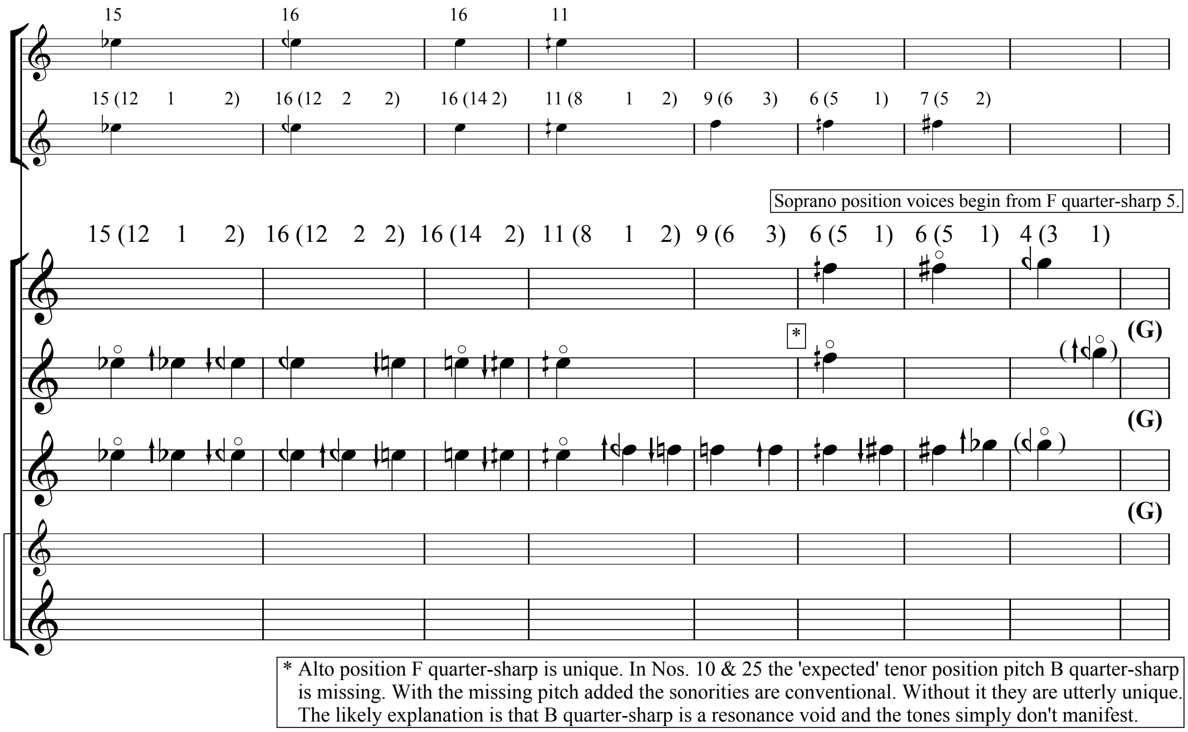
Figure 4.12 (Continued)

Having noted the distinct roles played by bass, tenor, alto, and soprano positions in the prototype sonority, and the further presence of a unique collateral resonance zone, it useful to redevelop the scalar levels of Figure 4.5 bringing all this data together. Figure 4.13 presents the scalar levels of multiphonic space noting where each voice position begins and ends, and precisely which tones each layer has. (This also reveals tones belonging to multiple positions.) Additionally, a harmonic symbol is placed over notes that sometimes appear as a difference tone in that specific register. If a note *exclusively appears* as a difference tone in that register, the notehead is further placed in parentheses, stressing the tone is never independent in that position. Accented tenuto marks indicate which pitches can appear in small dyads independent of any higher voices (see Figure 4.13 below).

Two continuums of timbral change have been identified above: the inharmonicity dimension and the instability dimension. There are surely additional dimensions of timbral change regards multiphonics. However, having charted out scalar steps in those continuums, it is possible to represent the timbre-space paths of these multiphonics in a two-dimensional timbral array. The multiphonics of *Radial* are organized at that level. The instability dimension for *Radial* was defined somewhat informally according to five steps. Multiphonics with three collateral tones are the stable level 0; those with two collateral tones are level 2; subtones are level 4. The odd levels 1, 3, and 5 are used to designate rougher members of each of those sets. The inharmonicity and instability dimensions are given as a timbral array in Figure 4.14.

Figure 4.13: The scalar levels of alto saxophone multiphonic space with voice position ranges and independence markings. Shows scalar level *e* distributed across the voice position ranges; the roles of difference tones and the collateral resonance zone are illustrated through harmonic signs (noting pitches that are sometimes a difference tone in that position), parentheses (noting pitches that are always a difference tone in that position) and accented tenuto marks (pitches that sometimes appear in low dyads without upper voices.)

Figure 4.13 (Continued)



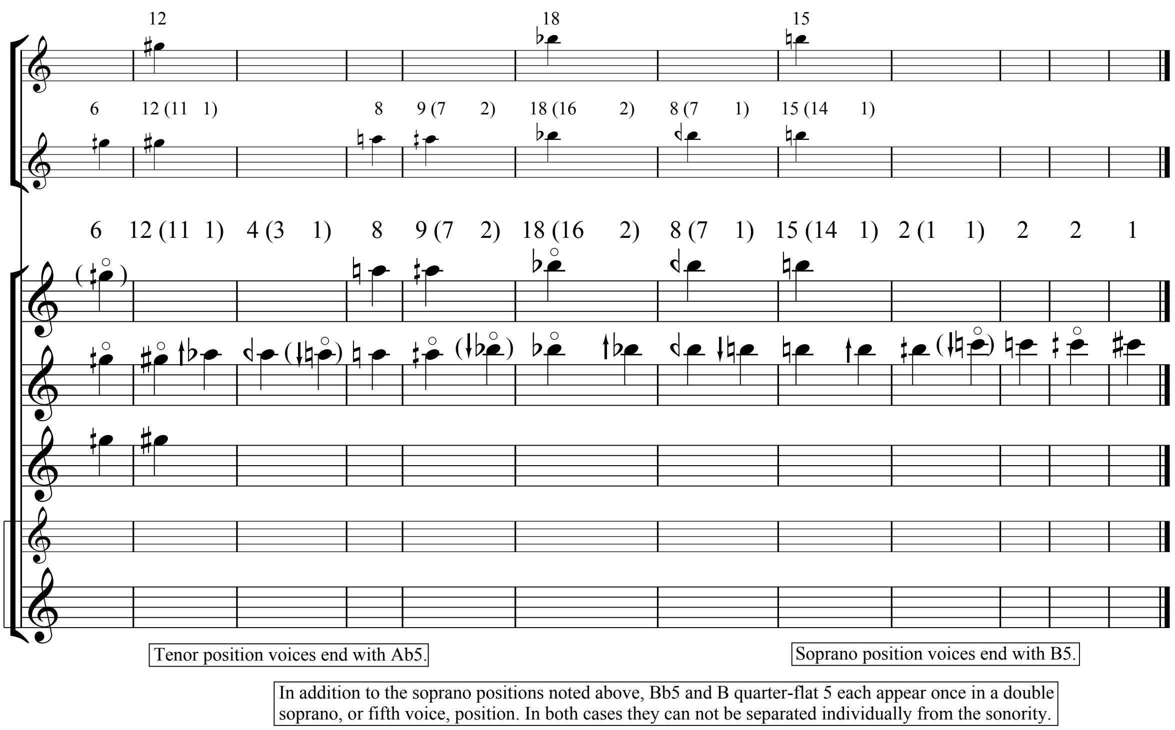


Figure 4.13 (Continued)

Di 4						
Di 3						
PrDi 2						
PrDi 1						
PrDi 0						
Pr 0						
PrCm 0						
PrCm 1						
PrCm 2						
Cm 3						
Cm 4						
Cm 5						
Cm 6						
	B/Instb 0	B/Instb 1	B/Instb 2	B/Instb 3	B/Instb 4	B/Instb 5

Figure 4.14: Alto saxophone multiphonic timbral space. The vertical dimension shows steps in the dimension of inharmonicity, which move out from the prototypical center. The horizontal dimension shows steps of instability that move out from the prototypical point of stability at left to greater instability at right.

In closing, we note that steps of inharmonicity and instability have been carefully charted out above, but significant mention has not been made regarding the levels of transposition at which these occur. Only limited portions of timbre space are available at each level of transposition and this is one factor that makes the system interesting. To look more closely at how timbre space is laid out over multiphonic space, it is easiest to look directly at the harmonic organization of *Radial*.

Before moving on to that study we must quickly introduce a final wider category in multiphonic space, called "resonance range." Each member of scalar level *e* has a particular resonance depth (Figure 4.4), a specific orientation in the voice position ranges (Figure 4.13), and a unique footprint in multiphonic timbre space (Figure 4.14). Collectively, we can call these factors the "resonance range" of each tone in multiphonic space. This larger multidimensional category embraces the material variety of all multiphonics contributing to resonance at particular scalar positions. What is intriguing is that, while complex, resonance ranges have coherent shape, and their interrelations point to still larger trends.[2]

Radial seizes upon resonance ranges as sound-mass entities, exploring their intrinsic qualities, unique differences, and the large-scale trends that unite them. The following analysis will show that its macro-form is a wedge-shaped progression of resonance ranges gradually coming to rest in resonance range Db4—arguably the most structurally relaxed resonance range in alto saxophone multiphonic space. Resonance ranges of this wedge-

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[2] For instance, it has been shown (Section 4.1.1) that resonance depth, which is one parameter of resonance range, is not arbitrary from position to position; it takes part in collective trends illustrated by the wide peaks and valleys of the resonance curve in Figure 4.4. Similarly, if we move from left to right in scalar level *e* observing the footprints for each tone in timbre space, other large-scale trends emerge—which also relate to orientations within the voice position ranges. For instance, bass positions at either extreme of the bass position range (F quarter-flat 3 to G4) produce unstable sonorities in high percentages (i.e. have footprints centered in the right half of timbre space); in addition to this, the upper region of the bass position range exclusively produces highly compressed sonorities (i.e. has footprints in the bottom half and in the right half of timbre space). Therefore values in the three dimensions of resonance range often exhibit degrees of interdependence, which develop across adjacent resonance ranges. The harmonic organization of *Radial* provides one broad example of how timbre space, voice position ranges and resonance depth interact across alto saxophone multiphonic space.

shaped structure appear vividly in the foreground of the work. Each of them extends from a structural series of bass position tones that elaborate Db4 (which is discussed in detail in Section 4.2.) However, powerful resonance ranges of middle and upper position voices (and indeed their opposites, the resonance voids) also influence the work, but their force is felt elliptically in the shape and harmonic structure of the prolongational background (discussed in Section 4.2.4).

As will be seen, *Radial* rarely presents resonances ranges in their entirety. Instead it works with groups of two to five multiphonics, which are representative of the traits and proportions of the larger entity. This reduction provides time to explore individual multiphonics and keeps relations between resonance ranges nearer to the foreground. The process of reduction is reviewed in greater detail in Section 4.2.5. For the curious, a larger work that explores the full density of resonance ranges is just around the corner.

4.2 The Basic Space at Work in *Radial*

4.2.1 *Radial*'s structure and harmonic background.

Alto saxophone multiphonic space informs the surface, middle ground and large-scale form of *Radial*. At the surface, the alto saxophone solo constantly projects multiphonic sounds that help us interpret the dense and slowly evolving harmony of the orchestra; at the same time the three digital delays of the orchestra project rich but elaborative groupings of the orchestral mass that could considered as the timbral equivalents of passing harmonies (or non-harmonic tones.) Still at the surface, though moving into the middle ground, the saxophone solo and its digital delay unfold the wider resonance ranges of individual notes, which link into a slow contracting harmonic progression. This harmonic progression, which defines the large-scale form of the work, is itself a layered presentation of three paths through alto saxophone multiphonic space; hence the tight coordination of material and structure.

The three simultaneously unfolding harmonic and timbral layers all prolong the relaxed resonance range of Db4, which throughout the composition is generally represented by two stable prototype multiphonics with bass position resonance at Db4. One is the harmonic series-based multiphonic with bass position Db4, the other is the [PrCm0] collateral multiphonic No. 83, with bass position C quarter-sharp 4 that bifurcates into resonance at Db4—a mere 7 Hz "step" away. The latter multiphonic is one of the three most striking and stable "alternate bass position" multiphonics discussed earlier in Section 4.1.3.[3]

Harmonic series-based multiphonic Db4 and [PrCm0] collateral multiphonic No. 83

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[3] The other two "most stable" cases flank the version on C quarter-sharp 4/Db4. One is found in the powerful resonance peak a quartertone below (C4), and the other is found in the powerful resonance peak a whole-step above (D quarter-sharp). (The larger role played by these peaks is discussed in Section 4.2.1.) To emphasize further the special and stable resonant strength of Db4 we note that of the full set of eighteen "alternate bass position multiphonics," discussed in Section 4.1.3, six can be immediately separated from the "more stable set" because their bass positions manifest in rough three-tone clusters (m. 7, Figure 4.10 gives one example); four more can be removed because their prototype inharmonicity lies outside the wider central region "PrDi0—Pr0—PrCm0" (see Figure 4.6). This leaves eight members in the "more stable set," four of which bifurcate into Db4 resonance (all shown in m. 3-6, Figure 4.11); the other four are distinct, bifurcating into resonances at D three quarters-flat 4, Eb4, E quarter-flat 4 and E4 respectively. Finally of these eight, there are the "three most stable" cases, with bass positions C4, C quarter-sharp 4 and D quarter-sharp 4, bifurcating into resonances at D three quarters-flat 4, Db4 and Eb4 respectively. These have already been discussed in detail above and are seen in m. 1-3 of Figure 4.11.

both sound clear variants of Db4, Db5 and Ab5, and these notes are always present in the score in some form. Both multiphonics are shown in Figure 4.15. The excerpt comes from time bracket 8:20 - 8:40 of the alto saxophone solo. It is a principal closing figure of the score.

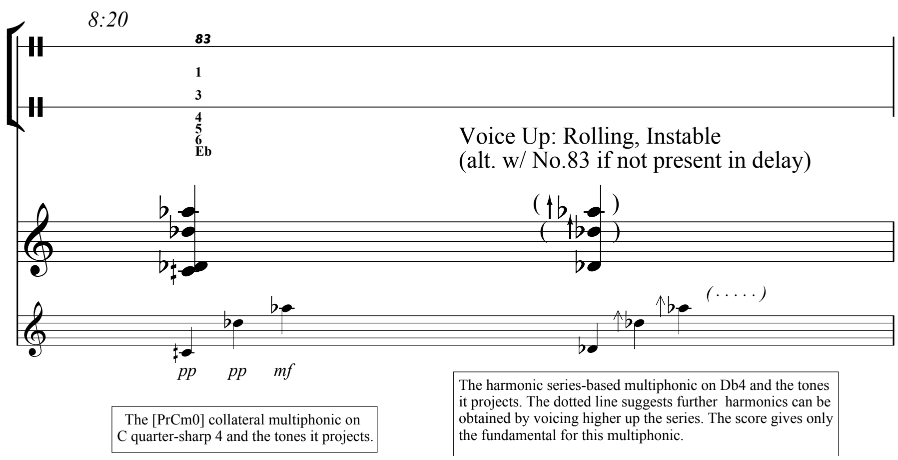

Figure 4.15: Resonance range Db4 in time bracket 8:20 - 8:40 of the alto saxophone solo. Shows vertical content of the harmonic series-based multiphonic on Db4 and [PrCm0] collateral multiphonic on C quarter-sharp 4 (No. 83). The composite sonority of both these multiphonics is prolonged throughout the work. In the score harmonic series-based multiphonic Db4 is notated by it's fundamental (alone) and the indication "voice up: rolling, unstable". The upper voices of the sonority are added here for clarification.

It might be noted that prolongation and resolution into one, or the other, of these two multiphonics would be more relaxing than prolongation and resolution into both. The preference for the latter case is one of the basic technical and aesthetic conditions of the piece. *Radial* does not principally seek to sound out individual multiphonics; it characterizes and sounds out the broader resonance ranges of its large-scale structural tones. The resonance range, arpeggiated by its multiphonics, is the basic entity of the composition.

Resonance range Db4 is a medium sized group of sonorities (depth 7); grounded exclusively in bass positions or alternate bass positions on Db4; with high harmonicity and low instability. All three features make it the most stable region in the composition (and alto saxophone multiphonic space.) Although presenting just one of the region's multiphonic sounds wouldn't capture the relaxed relations of the group, the stability of these two multiphonics and the slight friction created between them is largely characteristic of the whole.

4.2.2 The three harmonic and timbral strands.

The harmonic and timbral strands that elaborate the pitches Db4—Db5—Ab5 are a descending harmonic strand, an ascending harmonic strand, and a middle harmonic layer, called the centricity-fusion strand. The descending harmonic strand and ascending harmonic strand work together to elaborate the pitches Db4—Db5—Ab5 through a slow nine-stage progression of wedge-shaped or "double leading-tone" harmonies. They converge on these pitches at the close of the work and their opening harmonies present elements of these sounds in higher and lower octaves. The middle layer, the centricity-fusion strand, never strays from the pitches Db4—Db5—Ab5 by more than a quartertone in either direction. The first three events in the alto saxophone solo introduce each layer. By presenting them in the order seen below, the initial statement also projects the large-scale form of the work in microcosm (Figure 4.16).

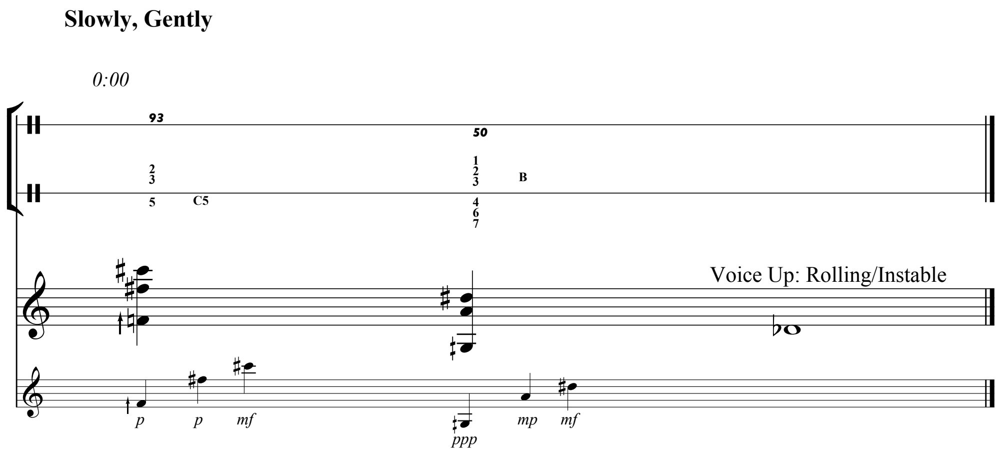

Figure 4.16: The three harmonic strands in time bracket 0:00-0:20 of the alto saxophone solo. Introduces the descending harmonic strand (bass position F4), the ascending harmonic strand (bass position G quarter-sharp 3) and the centricity-fusion strand (Db4—Db5—Ab5). This first wedge-shaped phrase is a microcosm of the large-scale form.

The composite progressions that result from the unfolding of these strands have some familiar if distant parallels from tonal and quasi-tonal contexts (Neapolitan/Dominant cadences and Phrygian/Lydian chromaticism) and more contemporary grid-based structures (chromatic, quartertone or hertz-based), which have been used to bring various structures into restful unison states. But the progressions of *Radial* are also quite distinctive because they ultimately derive from the shifting verticalities and colors of neighboring multiphonics with bass positions gradually merging at strong resonances around Db4—Db5—Ab5, which along with other gravitational partners detailed below, directly influence the sonic forces around them.

The behavior of harmonies and colors in *Radial* and the relaxed nature of resonance range Db4 itself, surely extend—at least in part—from the broad influence of the alto saxophone's fundamental, Db3, according to which Db4, Db5, Ab5 are the second, fourth and sixth harmonics. This oblique orientation on the saxophone fundamental is not the only centric-harmonic orientation that can be explored in, or supported by, alto saxophone multiphonic space. However, the perspective it creates certainly has a deep physical basis grounded in the essential dimensions of the saxophone's body—*including its length and conical rate of expansion*—and the many interventions upon the body, both structural and superficial—*such as the disposition and sizes of keyholes and the happenstance fingering limits of traditional keywork*—that make it the versatile musical instrument we know today. In this sense, *Radial* is a sonic and structural portrait of the device.

4.2.3 The ascending and descending harmonic strands.

The quickest way to introduce the resonance ranges of the ascending and descending harmonic strands is to look at the range of their bass motions in the bracketed area on the first page of Figure 4.4. It encloses a very particular region of alto saxophone multiphonic space, much of which is vividly drawn by the curve of Figure 4.4. The culmination point of both lines, Db4, is in the center of the bracket. The ascending strand moves up to this point from G quarter-sharp 3 (just inside the left bracket) and the descending strand moves down to the center point from "high" F4 (just inside the right bracket.) Both lines pass through curious symmetries and asymmetries.

The culminating point of Db4 rests in the middle of the two most powerful resonance peaks (C4 and D quarter-sharp 4) in the bottom half of alto saxophone scalar space. (A powerful double-leading tone motion to Db4 seemed inevitable from early stages of composing of *Radial*.) Significant secondary peaks flank the outsides of each of these (A quarter-flat 3 on the lower side, and E4 on the upper side.) Thus ascending and descending harmonic strands both pass through two powerful resonance peaks on route to Db4, with the last peaks—in both cases—being the most powerful. However, in contrast to this basic symmetry, the peaks themselves have an *asymmetric harmonic disposition*. This was forecast earlier in the creation of "pentatonic" level *d2* (Figures 4.5 and 4.12) with the segment including "A3—C4—D4—E4."

This symmetric/asymmetric organization of color is more pronounced when considered from level *e*. Beyond presenting an asymmetric positioning of similar forces, it also presents an asymmetric positioning of *differing forces*. This is because level *e* includes the powerful peaks of level *d2*, but it also includes midrange peaks, valley regions, and a number of resonance voids—each with their own unique positions. As discussed in Section 4.1.1, these resonance voids also carve out different size steps, so symmetry/asymmetry is also expressed at this level through the varying distances of adjacent step sizes.

Consequently, the ascending and descending strands, as bracketed off in Figure 4.4, each have nine members (at quartertone resolution), but because the ascending strand contains two "large steps"—via resonance voids—it ends up covering more ground than the descending strand (a diminished fifth in eight steps as opposed to a major 3rd in eight steps). Other asymmetries ensue because the nature of the scalar level changes radically when entering its second octave (Db4). Thus, for instance, the descending line has a richer surface elaboration at the eighth-tone level. All of this is clearly detailed in Figure 4.17

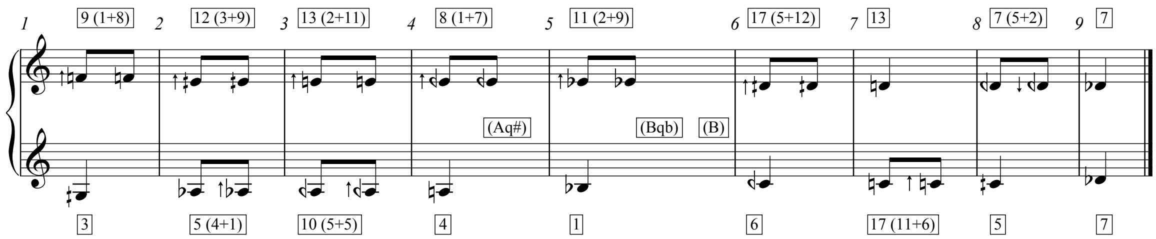

Figure 4.17: Ranges and bass positions of the descending and ascending harmonic strands converging on Db4. Each strand has nine members but the ascending strand covers more ground. Boxed numbers show how resonance depth for each pitch; the brackets provide the same information at the eighth tone level.

Only a few changes were needed to move from the pre-compositional sketch of Figure 4.17 to the actual selection and progression of bass positions observed in *Radial*. Figure 4.18 shows the large-scale movement of bass positions in both strands as seen in the score. At the surface, these tones are elaborated by up to five multiphonics. For the sake of clarity, that detail is shown later. Also not shown at this stage is the way these lines form a single compound line in the surface of the saxophone solo.

The most important interventions are in the ascending line, where two scale tones are placed into m. 4 and three scale tones are spread out over the last four measures. Both of these moves refine the large-scale structure of whole, forming two halves punctuated by powerful cadences. (They also smooth out the temporal presentation of resources in the ascending line, which is a concern owing to its greater variation of densities.)

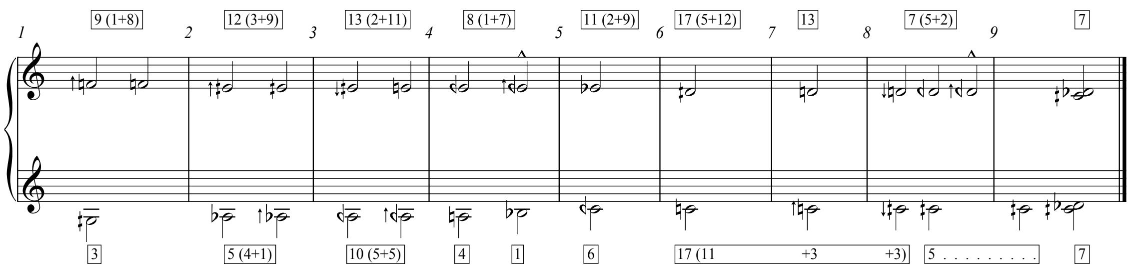

Figure 4.18: Bass positions of *Radial*'s nine-stage progression of double leading-tone harmonies. Each measure corresponds to one minute (three time brackets in the score). In the ascending line two scale members occupy m. 4 and three scale members are spread across the last four measures. Cap accents in the descending line point to small *rising steps* that break the trend of downward motion. Boxed numbers show resonance depth; brackets give that information at eighth tone level. The single tone in m. 6 of the descending line indicates that available eighth tone variation for D quarter-sharp 4 (seen in the boxed figure above the staff) is not exploited. In all other cases such variation is explored.

Note the descending line has two small "ascending steps" marked with cap accents. These "turns" in the bass position allow more prominent descending continuities to develop in the upper voices of their multiphonics. From this perspective it is hard to say if these events are artifacts or interventions. However, their importance is *underscored* by bringing the movement of the ascending line into an alignment wherein both lines punctuate the division of the whole into two large halves.

Before moving on to more complex issues it should be pointed out that all significant peaks and valleys seen in the Figure 4.4 come into play through the middle and upper voices of the bracketed bass positions under discussion. We will see that powerful peaks such as

Ab5, Bb5, and B5 and the resonance void at G5 exhibit clear effects on the harmonic progressions of *Radial*. Ultimately *Radial* covers the full range of sounds presented in the curve. The upper limit of C#6 is sounded as an upper voice of F4 in the first time bracket of the work. The lower limit of F quarter-flat 3 also appears in the work, during time bracket 1:20-1:40, through a moment that temporarily reinterprets the bass note F quarter-flat 4 as a tenor voice to F quarter-flat 3.

Finally, Figure 4.4 cannot reveal the full nature of this particular selection of resonances ranges—after all it directly depicts only one of their three dimensions. The lower and upper limits of the ascending and descending lines (G quarter-sharp 3 and F4, respectively) were not chosen simply to make equal 9-member scale segments merging at Db4. (The difference of step sizes between the strands, for instance, would still be noticeable even if the descending line had 10 or 11 scale members instead of 9.)

The precise upper and lower limits of these strands were chosen because they are *limits in the transpositional registers of stable and central prototype sonorities*. Going higher or lower than either of these points creates "timbral gaps," wherein particular scale members lack any central and stable prototypes to choose from. Specifically, going lower than G quarter-sharp 3 creates gaps with no "PrDi0—Pr0—PrCm0 + B/Instb 0" multiphonics; and the choices only tend to become more restricted as the bass position falls further.) Going above F4 is more problematic. For instance, between F quarter-sharp 4 and the last bass position G quarter-sharp 4, only one bass position sonority manifests with an alto voice (not to mention a soprano voice), and that case is a highly compressed [PrCm2]. Indeed, all prototypes at the upper limit are highly compressed *and* (excepting the one case just mentioned) without alto voices.

As it stands all nine resonance ranges, of both harmonic strands, have at least one stable prototype multiphonic to choose from, which does appear in the compositional surface. The unstable resonance range of Bb3 (see m.5, Figure 4.17), which is comprised of a single [B/Instb 5] non-prototype subtone, is an exception to this. However within the score, it is folded into the end of Section 4 (resonance range A3; see m. 4, Figure 4.18) and associated with that region. (Nonetheless it receives large and appropriate structural function, discussed below.) Therefore each of the nine areas of *Radial*, in both harmonic strands, presents at least one path into, or one path out from, the most central and stable timbral region ("PrDi0—Pr0—PrCm0 + B/Instb0") of alto saxophone multiphonic timbral space (Figure 4.14) This accounts for some of the cyclic character of the work.

Seeking to prolong the essential quality of resonance range Db4, *Radial* has implicit limits to the materials it selects. Its resonance ranges have shared traits, but the work also explores the unique materials of each resonance range in the group. It's time to look more closely at the timbral varieties and inharmonicity levels present in the score and their structural disposition.

4.2.4 Inharmonicity in the ascending and descending harmonic strands.

The spatial layout of timbral inharmonicity is a thorny subject quickly elucidated, in general form, by elaborating Figure 4.18 to show the *outer intervals* of both harmonic strands in the basic wedge-shaped structure. The results (Figure 4.19) are surprising. Bass positions of the descending line strictly descend throughout (except for the two small cadential "turns" with cap accents); but its upper voice follows three separate melodic descents. The first travels from C#6 to A quarter-sharp 5 over five sections (five minutes in the score). The second steps back to B5 and then travels down to Bb5 over two sections. The third and final descent steps back up again to B5 and then over two sections travels to A quarter-sharp 5 once more, before finally "popping" down to Ab5, its final resting point.

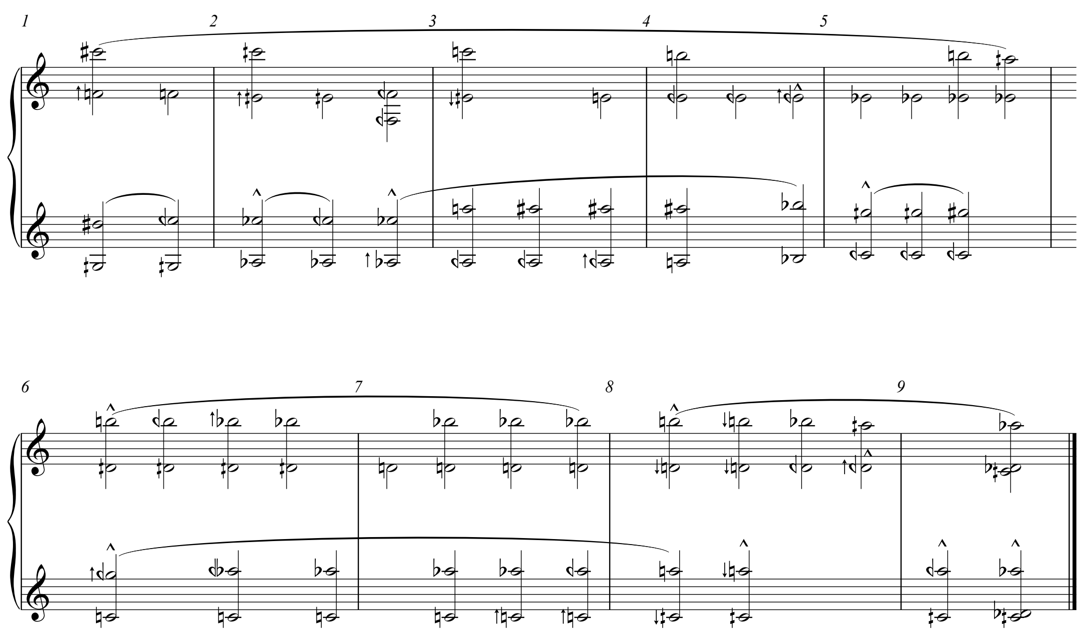

Figure 4.19: Surface reduction of the outer voices of the ascending and descending harmonic strands. Shows the bass position and upper position voices of each multiphonic used in each section of the score. Missing upper voices indicate the multiphonic in question has no alto (upper position) voice. Cap accents indicate a step motion contrary to the general trend.

It is as though a centrifugal force pushes sounds away from Ab5 until, at last, a centripetal force finally pulls them in. This notion is actually clearly charted in Figure 4.4 by the steep resonance valley at A quarter-flat 5 (the "centrifugal force") and the welcoming powerful upper peaks at B5 and Bb5, and the final narrow peak at Ab5 (the highly selective "centripetal force.")

The upper voice of the ascending harmonic strand also takes Ab5 as its ultimate resting point. However, its long-range path is even more eccentric. The outer voice of the ascending strand rises to Ab5 through five separate ascending phrases and finally descends upon it in three downward steps. The first two small phrases from Eb5 to E quarter-flat 5 reflect the power and girth of the peak at E quarter-flat 5. In the next phrase the upper voice leaps up, gradually pulled in to Bb5, the highest peak of the curve. (Oddly, in some of those chords this outer voice sits on top of a three-note sonority, while in others it sits on top of a four or five-note sonority, but all cases, it is *the outer upper voice*.) The next small phrase targets Ab5 early on, approaching it again from below, now via G quarter-sharp 5. The motion is repeated in the fifth phrase, but from a lower starting point. (The wide interval "G quarter-flat 5—A three quarters-flat 5" results from a skip over the resonance void a G5.) The fifth phrase then lands on Ab5 once more, but continues, moving beyond it A5. At last, the outer voice steps down to "low A5" and then A quarter-flat 5 before landing once more on Ab5, the final position in the score.

In fact, that final descent is something of a compositional artifact—the members of the resonance range could be ordered to rise from Ab to A—but even if the order of the last three notes is reversed the larger lesson remains. Movements or "steps" in the continuum of inharmonicity are steered by powerful resonances, which could be likened to gravitational pedal points or pedal "clusters." (This was part of the basic intuition laid out at the start of chapter 4.) Thus change in inharmonicity is often the result of *something staying the same*. A simple example can be taken from sections 1 and 2 of the ascending line in Figure 4.19. In section 1, G quarter-sharp 3 has two levels of "bass-alto" inharmonicity available via Eb5 and E quarter-flat 5. In section 2 the bass position rises to Ab3. Two levels of "bass-alto" inharmonicity are still available to that note, but it is through *the same two previously available pitches*, Eb5 and E quarter-flat 5. Thus the level of inharmonicity becomes more compressed—*through* the alto position notes staying the same.

If we take the tenor voices of these multiphonics into account, the multiphonics from section 1 turn out to be [Pr0] and [PrDi0] respectively, while the section 2 multiphonics turn out to be [PrCm0] and [PrCm2] respectively. Therefore the bass positions move up by one quartertone from one section to the other, while the tenor voices do not rise with them, or stay the same, but actually compress further—and somewhat diversely. Tenor voices in ([Pr0 and [PrDi0]) create a step in timbral space, while the tenor voices of [PrCm0] and [PrCm2] constitute a skip.) None of this, however, is registered in the alto voices, which simply stay put on Eb5 and E quarter-flat 5 throughout. That behavior certainly accounts for the broad and powerful peak on those pitches in page 2 of Figure 4.4.

There are, of course, exceptions to these broad trends and correspondences. Choosing very selectively, one could present different kinds of "inharmonicity developments" with the resource of the space, but these would be set against the background trend. One final example illustrates the comprehensive influence of "the background trend."

The "jumpy" character of the upper voice in the ascending line of Figure 4.19 could be attributed simply to the resonance void at G5 "limiting" materials and forcing approaches to Ab5 from both above and below the pitch. That is surely reasonable, but it is only part of the story. The larger picture suggests the whole downward slope from E5 to G5 contributes to the shifting character of upper voice. For instance, we could imagine E5 to G quarter-flat 5 as "an area of powerful peaks", wherein the upper voice would simply recycle through the richness of the area, until at last, it "hopped" over the void at G5 and landed on Ab5. This might seem merely hypothetical, but it is *exactly what happens to* the upper voice of the descending line. It retreads materials from the high peaks of B5 to A quarter-flat 5 until it finally "hops" over A5 to arrive on Ab5. The curve (Figure 4.4) doesn't show A5 as a resonance void—in the figure it is part of a downward slope—but it never appears in relation to any bass position above Db4. Its absence in the upper voice of the descending line is not a compositional artifact; it is, in fact, a void relative to those materials, it is a fundamental aspect of the space. So it is not merely the void of G5 that makes the eccentricity of the outer voice of the ascending line, it is the broader context of the whole descent over six quartertones to that void. In the case of A5, however, moving six quartertones to the right of the pitch encompasses the entire final peak region of the space and still more.

Let's now explore a detailed example of how the multiphonics in Figure 4.19 were chosen and how their harmonies and timbres interact.

4.2.5 Harmonic and timbral hierarchies in the ascending harmonic strand.

Figure 4.20 demonstrates the process of confronting full resonance ranges for each structural bass position tone, highlighting its various characteristics, and forming a final representative selection that also emphasizes trends across the ranges. The top grand staff (marked "The Resonance Ranges") shows the total material for all five resonance ranges traversed by the ascending strand in the first four minutes of the piece (i.e. the first four sections in Figure 4.19). Double barlines separate the five resonance ranges. Within the resonance ranges multiphonics are organized first by rising eighth-tones in the bass positions, then by rising alto positions, and finally by rising tenor or soprano positions. (Most examples fit neatly in this scheme. However, m. 7, for instance, was seen as something of an outlier to the organization and simply placed last amongst examples at that bass position; it could be placed before the sonority at m. 4 to no noticeable structural effect.) The smaller staff in the grand staff shows which tones can be separated from each multiphonic, and is thus a broad measure of the instability continuum.

Placed in this way a general upward trend is observable, but it's still a motley group. A form begins to emerge however, by separating the most stable sounds [B/Instb0-1] from the less stable sounds [B/Instb2-5]. The staff marked [B/Instb0-1] gives only the most stable sounds from each resonance range. By simply dropping them down from their position in the grand staff, their position in the sequence is retained. The staff is more workable, but its

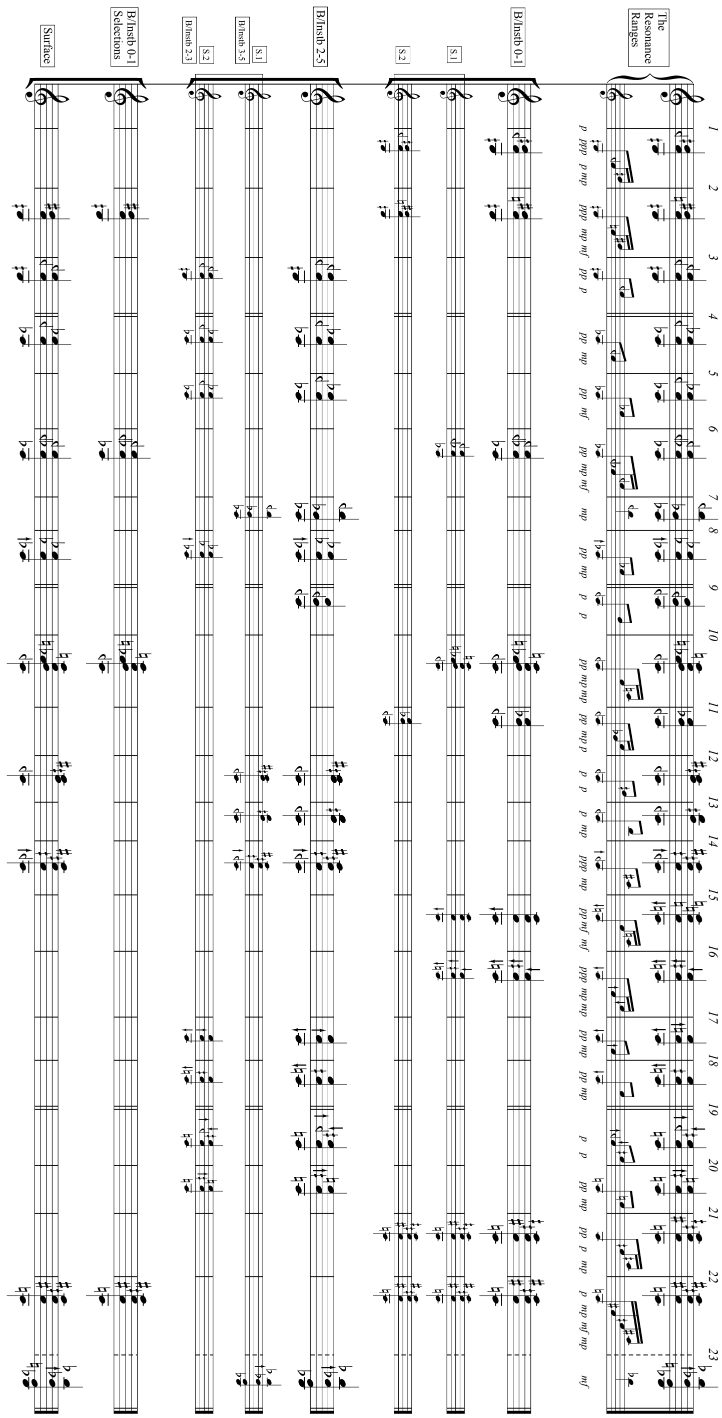

Figure 4.20: Process from resonance range to surface. The first five resonances ranges, the interior organization of their materials, and the surface formulation of the first four sections (four minutes) of the ascending harmonic strand.

precise features are still unclear. Following basic principles of voice-leading and the general concept of streaming outlined by Bregman (1990), two further "streams" can be discerned. The first stream (S.1) is placed immediately below larger group; the second stream (S.2) is placed below S.1. The first stream charts a gradual dilation of bass-tenor intervals from an initial highly compressed state [PrCm2]. The group also quickly branches out into wider sonorities. The second stream follows a compressed alto-tenor position tritone that settles over a dilated tenor-bass [PrDi0] and finally joins with the terminal sonorities of (S.1) through common tones and chromatic (actually microtonal) voice leading.

The same basic process is followed for the less stable multiphonics. First a staff is created with all the less stable sounds [B/Instb2-5], and two more focused streams (S.1) and S.2) are derived from it. The organization of these streams goes somewhat further by also dividing materials according to finer degrees of instability, creating a "most unstable" timbral stream (S.1_B/Instb 3-5) and a "less unstable" timbral stream (S.2_B/Instb 2-3).

To create the actual surface selection, choices were first made from the stable streams, forming a clear progression of the most stable sonorities. The "stable selection" staff [B/Instb0-1_Selections], near to the bottom, shows that choices began on (S.2) but then moved to and stayed on (S.1). Finally [B/Instb0-1_Selections] was elaborated by forays into the two streams of unstable sonorities: first (S.2_B/Instb 2-3) and then the tenser (S.1_B/Instb 3-5). This produces the bottom staff (marked "Surface"), which is the ascending harmonic strand as it appears in the first four minutes of *Radial*. Talking through this process reveals the larger purpose of the organizational scheme depicted in Figure 4.20. The goal is not simply to find and articulate streams, but rather to isolate clear trends within and beyond the resonance ranges and then choose when and why to move between them.

At the close of Section 4.1.4 it was remarked that the "strength" of powerful resonances would support a variety of timbres but that these sonorities, owing to the continuity of powerful resonances, would also exhibit some form of harmonic prolongation. This is what is seen in the final staff (marked "Surface") of Figure 4.20. Movements from stable to stable timbres, shown alone in [B/Instb 0-1_Selections], are distinct enough to create "progression" in the sense of branching structure tensing and relaxation. However, the *timbral progressions* explored by steps into the unstable streams "S.2_B/Instb 2-3" and "S.1_B/Instb 3-5" largely produce *weak harmonic prolongations*. The great exception is the final unstable multiphonic in Figure 4.20 on bass position Bb3. (This is the only sonority in resonance range Bb3, which sits between two resonance voids.) Although Bb5 prolongs into its upper voice, bass movement is pronounced, both to and from the chord, and the structure of the sonority itself—a diminished voicing with slight microtonal inflection—is totally unique. This non-prototype subtone is also tense [B/Instb 5]. Therefore it is a strong harmonic *and* timbral progression; these features ultimately place it quite high in the overall branching structure of the work.

Figure 4.21 gives a branching structure for the final staff ("Surface") in Figure 4.19. (For orientation it also shows structural levels a-c for the second half of the ascending strand.) This was an impressionistically conceived compositional device, not an analytical rendering of the finished work, but it does demonstrate that the unstable multiphonics are

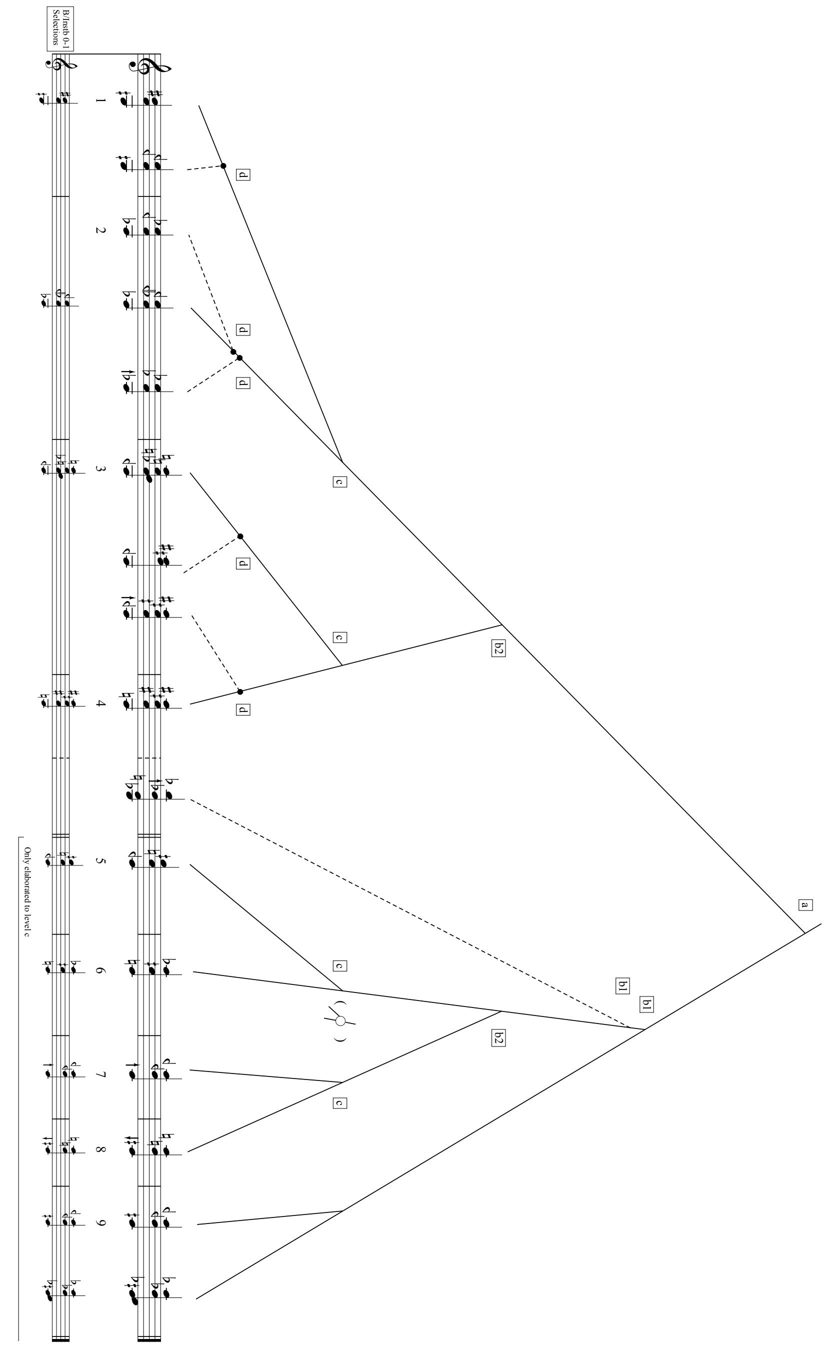

Figure 4.21: Branching structure for example in the final staff of Figure 4.20. Level a-c elaboration is given for the second half of the work. Dotted branching lines indicate the further presence of timbral progression. The bottom staff shows that the selection of stable sonorities (Figure 4.20) fills level c. Stable sonorities also fill level c in the second half of the work.

largely near-surface (level d) weak harmonic prolongations; however dotted lines moving to these sonorities indicate they are also “timbral progressions” in the context of alto saxophone timbral space (Figure 4.14). Level c has only stable multiphonics forming harmonic progressions and timbral prolongations. This relation is made clear by placing the original [B/Instb0-1_Selections] staff underneath the branching diagram. However, not all items in [B/Instb0-1_Selections] reach the same structural levels. Level b2 shows certain stable multiphonics build deep cadential patterns, which have a normative prolongational structure. Level b can also be interpreted as having a deeper structure, level b1, dividing the work into two halves. The prolongational basic form of level b1 is arguably the most salient of the two large-scale divisions and receives significant sonic reinforcement from the arrangement of orchestral color and the centricity-fusion strand, both of which are discussed briefly below. It is noteworthy that the prolongational basic form at level b1 is articulated by the non-prototype subtone [B/Instb 5] built on bass position Bb3. It is the only timbral progression (the only highly tense member of the instability continuum) to rise above level d—and it goes straight up to a defining role in level b1.

Figure 4.22 gives a similar but fuller branching structure for the ascending harmonic strand, with time span reduction levels (TSR) for each tree level. Dotted barlines in the staves underscore where bass positions are not precisely contained by the steady temporal boundaries of the nine large sections. Figure 4.22 shows that the second half of the ascending harmonic strand embraces two locations (at C quarter-flat 4 and C4) where level d

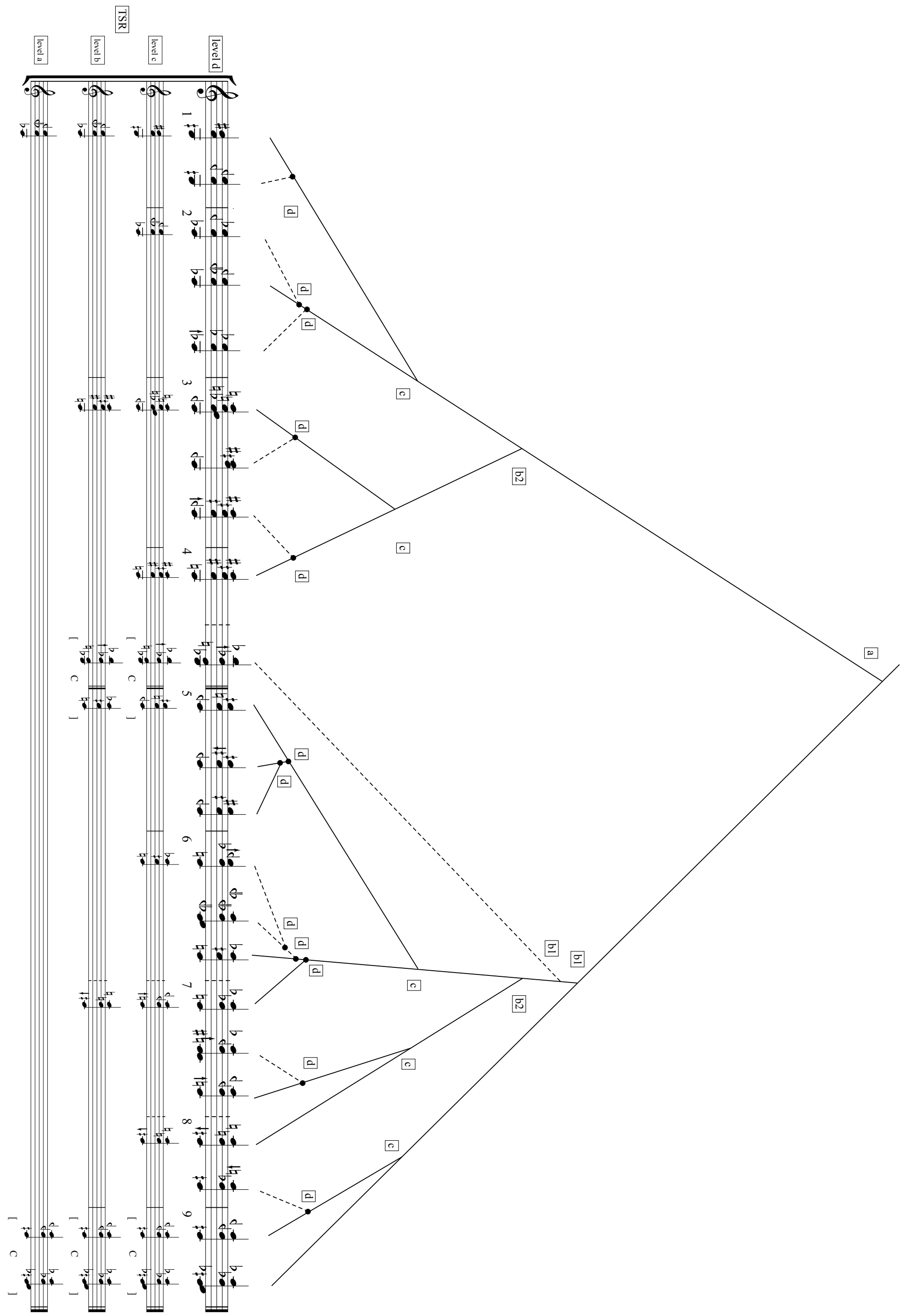

Figure 4.22: Complete branching structure for the ascending harmonic strand with time span reduction levels (TSR) for each level of the tree. Dotted branching lines indicate timbral progressions. Dotted barlines show single bass positions not exclusively contained by one of the nine the sections.

Is elaborated by *both harmonic and timbral prolongations*. (Stable sonorities that elaborate level *d notably lack* dotted branching lines.) These static adjacencies point to a feature of the material itself. Section 4.3.1 noted the outer boundaries of the ascending and descending harmonic strands were chosen because stable prototypes did not—or did not consistently—extend beyond these points. Beyond thinning out at these points, they also greatly increase in number with movement towards the center point of Db4. The ratio of unstable to stable multiphonics is generally 3:1, 2:1 or 1:1 in the outer halves of the ascending and descending strands (an extreme case is 9:1), but that balance reverses within the inner halves of each line — converging on Db4 — where typical ratios of unstable to stable multiphonics are 1:4 or 2:3, with more extreme cases being 1:6 or 1:8. The increase in stable harmonies towards the center point is therefore also a feature of the descending harmonic strand. Let's look more closely at relations between the two harmonic strands.

4.2.6 Harmonic and timbral hierarchies across the ascending and descending harmonic strands.

Figure 4.23 provides the complete content of all multiphonics used by the descending and ascending strands. The descending strand is in the top staff and the ascending strand in the bottom staff. The upper middle staff shows the stable sonorities [B/Instb 0-1] of the descending strand; the lower middle staff shows the stable sonorities [B/Instb 0-1] of the ascending strand. Stable sonorities significantly increase in the second half of both strands.

Figure 4.23: The complete multiphonics and basic timbral divisions of the descending and ascending harmonic strands. Shows the full content of all multiphonic sonorities in both strands and the large-scale organization of more and less stable timbres.

Figure 4.23 also shows that collateral resonance zone multiphonics regularly appear in the descending harmonic strand starting from section 1, but—with the exception of the tenor bifurcation in section 3 (of the ascending strand)—stable sonorities do not interact with the collateral resonance zone until the second half of the composition, where they become pervasive in sections 6, 8 and 9. As the lines merge their vertical structures and their collateral resonances stabilize. Consequently, their uniting creates a tense composite-force ringing sound that characterizes much of the second half of the work and contrasts the gentler sonorities of the composition's first four sections.

Although stable sonorities in the two strands have a broad collective shape, their precise locations are frequently contrasting. Consequently, it is evident that level d branching in the descending strand differs from that of the ascending strand. However, branching structure in both strands does coordinate at higher structural levels. Figure 4.24 (below) demonstrates this through a level b time span reduction (with branching tree representation) for the bass positions of both strands.

Figure 4.24 shows quasi-tonic prolongation at level a. Implied cadential 6-4 voicing at section 1 hints that "dominant" functionality has begun at the start of the piece, similarly the lingering C quarter sharp 4 of section 9 shows dominant voice-leading never fully resolves to tonic at the close of work. (It is as if the penultimate B naturals of Sibelius's Seventh never quite reached to C.) A quasi-dominant cadence at b1 divides the work into two halves; the cadential V at section 5 prepared by "diminished" harmony. The move from there (quasi "vii^{o7}/V") to V gives the only timbral progression in the upper structural levels. At level b2 movements from "cadential 6-4 to V" and "V to I" are embellished with passing harmonies, which in tonal contexts spawn mixed-mode Phrygian borrowings. (A tonal "cousin" work might read distant cadential 6-4 with deceptive motion to bVI; vii^{o7}/V to V, amplifying to V^{b5}; a microtonally-inflected V^{b5} double leading-tone cadence to I, which in final position is promoted up from level c; and not all cadence members move to I.) The background is elaborated differently here but, arguably, some kindred character or broad influence remains.

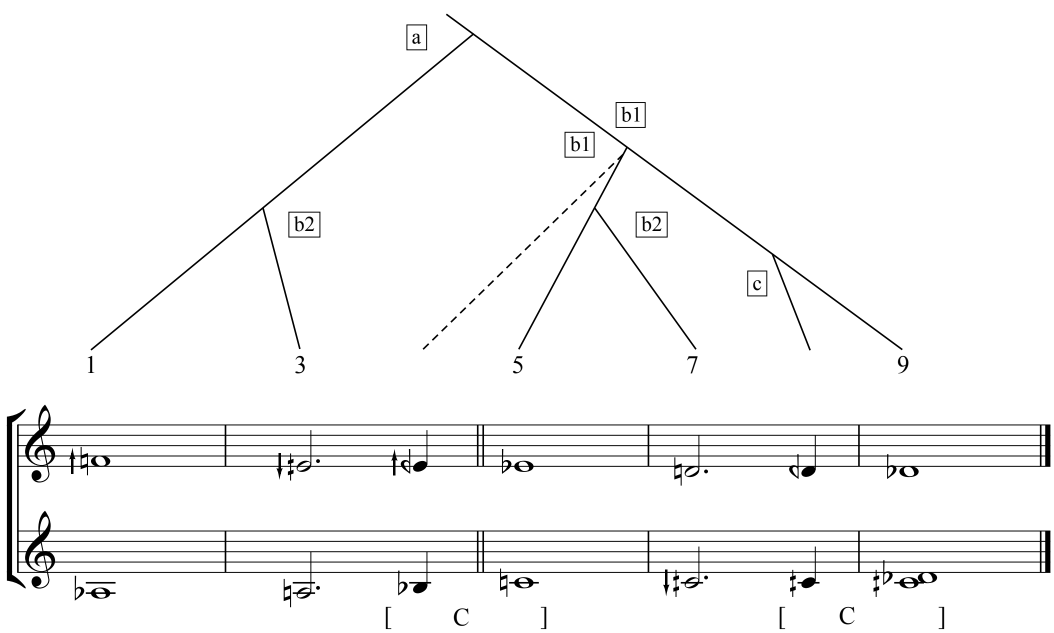

Figure 4.24: Level b time span reduction for the descending and ascending strands. Each strand is represented by its bass positions. The dotted branching line indicates a further progression is present through timbre space.

Little mention has yet been made of the centricity-fusion strand. However, when contemplating the salience of deep level structures, such as Figure 4.24, it is pertinent to remember that this third strand of harmonic activity is always present. The centricity-fusion strand never strays far from Db4—Db5—Ab5. Taking out the microtonal inflection and branching structure detail of Figure 4.24, we can informally recast the progression adding in the effect of the centricity-fusion strand (Figure 4.25).

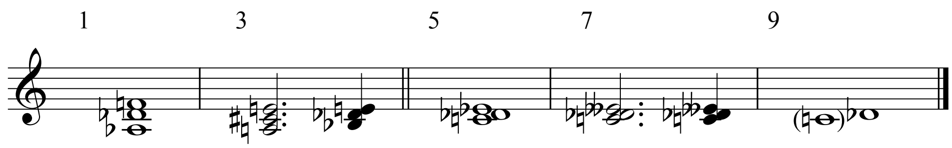

Figure 4.25: Informal "level b" time span reduction for all three harmonic strands. They are represented here by their bass positions alone.

The reduction in Figure 4.25 presents the deepest portrait of all three layers of *Radial'*s large-scale harmony. However, it presents only the "centricity" function of the centricity-fusion strand, and that role is also oversimplified because it constantly projects centricity around all three pitches Db4—Db5—Ab5. Let's look more closely at centricity-fusion strand and its dual functionality. Afterwards we'll briefly review surface timbral elaborations in the score, and close our analysis by observing how these materials form larger "timbral grouping structures" that reinforce the harmonic forms outlined above.

4.2.7 The centricity-fusion strand.

As discussed earlier the centricity-fusion strand also prolongs resonance range Db4. Indeed, it constantly projects variants of the notes Db4—Db5—Ab5, and therein lays its "centricity" function. Together with the alto saxophone solo's opening harmonic series-based multiphonic Db4, it anchors the opening complex sonority of *Radial* in a distant "tonic function" or, as mentioned above, a "dominant function overlaid with tonic derived suspensions (V^{6-5}_{4-3})." Near the close of the work it provides the harmonic anchor on which all other materials merge. Throughout the composition, like any traditional pedal point, it draws attention to early cadential arrivals or coincidental doublings upon these pitches. Cumulatively, these are the basic centricity functions.

The "fusion" function comes from a harmonic and timbral progression inside the strand that creates high roughness during central sections 4, 5, and 6 of the composition. This roughness appears again, *between the middle centricity-fusion strand and the outer strands*, in sections 7, 8, and 9 as the outer layers finely hone in on the closing pitches of Db4—Db5—Ab5. It is well established that increased roughness promotes perceptual fusion of component elements. This is a traditional role of orchestral percussion instruments and such added "noise" is also key to forming convincing entities out of additive frequency based reconstructions of a traditional instrumental timbres. Roughness in this strand performs that role, and it is created by the gradual rise and fall of tense beating patterns (groups of combination tones) within several critical bands spread throughout the frequency range of the composition. Figures 4.26 through 4.30 explain the derivation of the centricity-fusion layer and detail how it's fusion function works.

Figure 4.26 provides the prolongational background of the centricity-fusion strand and the first four stages of its elaboration. At the lowest level it prolongs the two key multiphonics of resonance range Db4, harmonic series-based multiphonic Db4 and [PrCm0] collateral multiphonic No. 83. The first elaboration is a temporal dilation of harmonic series-based multiphonic Db4 as performed by the solo alto saxophone.

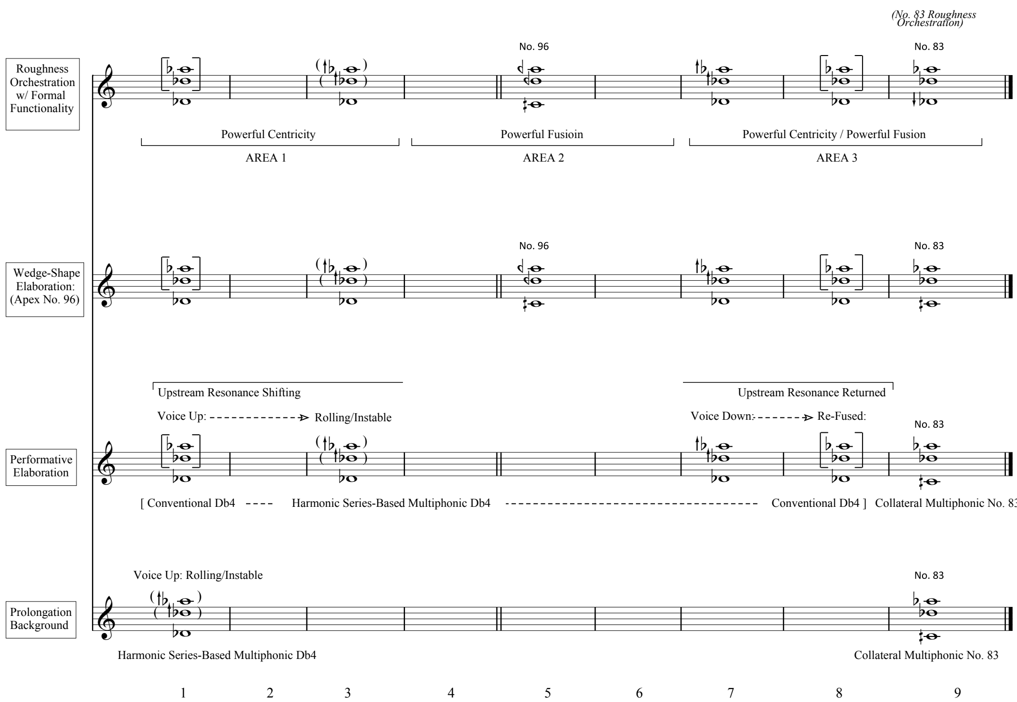

Figure 4.26: The prolongational background and perceptual functions of the centricity fusion-strand. The prolongational background is shown (at bottom) together with the first three stages of its elaboration and the rise of its large-scale harmonic and perceptual functions. Numbers below the prolongational background refer to nine major sections of *Radial*'s large-scale form.

The soloist's event envelope has three stages, wherein the performer plays the tone as a conventional timbre; shifts the voicing of the upstream resonance destabilizing amplitude distributions amongst the first upper harmonics thereby raising the frequencies; and returns the upstream resonance to normal position re-fusing the component elements of the sound. In short, the level presents “de-composition” via upstream resonance. The soloist does this gradually; the forces of the centricity-fusion strand do it more gradually still creating a large-scale associational (motivic) relation to that material. After exploring the upward and downward trajectory of the harmonic series-based multiphonic event envelope, the strand makes a subtle relaxation down into [PrCm0] collateral multiphonic No. 83.

In the next higher level, the temporal center of harmonic series-based multiphonic event is elaborated by progression into multiphonic No. 96, the nearest stable multiphonic outside of resonance range Db4. The progression gives the structure an arch form with an apex of high roughness. The top staff shows the functionality that arises from the process; all sounds presented there manifest in the compositional surface of the strand (and score) as landmarks in a more gradually drawn out harmonic process, which is discussed below. These basic materials, which establish the total ambitus of the centricity-fusion strand, are recounted by the final three multiphonics of the alto saxophone solo. (Figure 4.27)

Figure 4.28 synopsizes the “drawn out harmonic process” that engulfs the top line of Figure 4.26 and details the orchestration of the strand. Each note in the figure represents nine notes in the score, which typically repeat in groups of three. The notes of the score are “averaged” in Figure 4.28 simulating the general trajectory of the strand. It is worth stressing, however, that the composing out of those materials markedly resists this process. The tiny intervals of the centricity-fusion strand are laid out as a mix of skips (and steps) that create a fine “bristling” of unique adjacencies and verticalities in the delays and across the group. For instance, in the score, six consecutive microtones “0, 1, 2, 3, 4, 5” might be

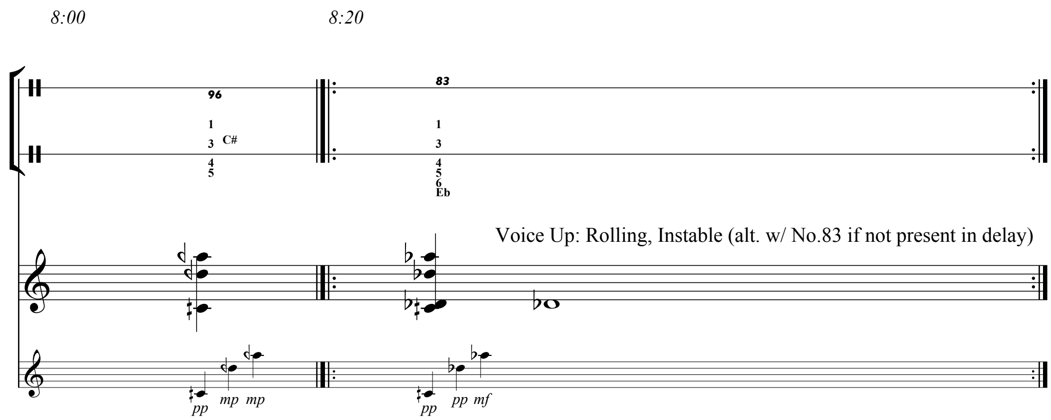

Figure 4.27: Materials of centricity-fusion strand prolongational background synopsized in time brackets 8:00-8:20 and 8:20-8:40 of the alto saxophone solo. The last three sonorities presented by the alto saxophone solo lie at the root of the centricity-fusion strand. The ambitus of the strand does not extend beyond the limits of these sonorities.

ordered as "0, 4, 1, 3, 2, 5" to produce intervallic variety and differing "trichords" that create a rich range of beating patterns and color variations within the microscopic vertical territory, and long-range temporal trajectory, of the strand. However, in Figure 4.28 those first three notes "0, 4, 1" would be averaged together as "2" and the next three notes "3, 2, 5" would be averaged as "3", thereby presenting only the slowly rising trajectory of the line. One final caveat, Figure 4.28 shows three instruments on the top staff; in the score these instruments participate in the strand alternately, they do not appear as doublings or triplings.

The small additional accidental steps of "+1, +2, +3" and "-1, -2, -3" seen in Figure 4.28 appear throughout the score in the centricity-fusion strand. They are common in the outer strands during the first half of the work, but are less so in the second half. These accidental signs split individual eighth tones into four fine steps of pitch or color.

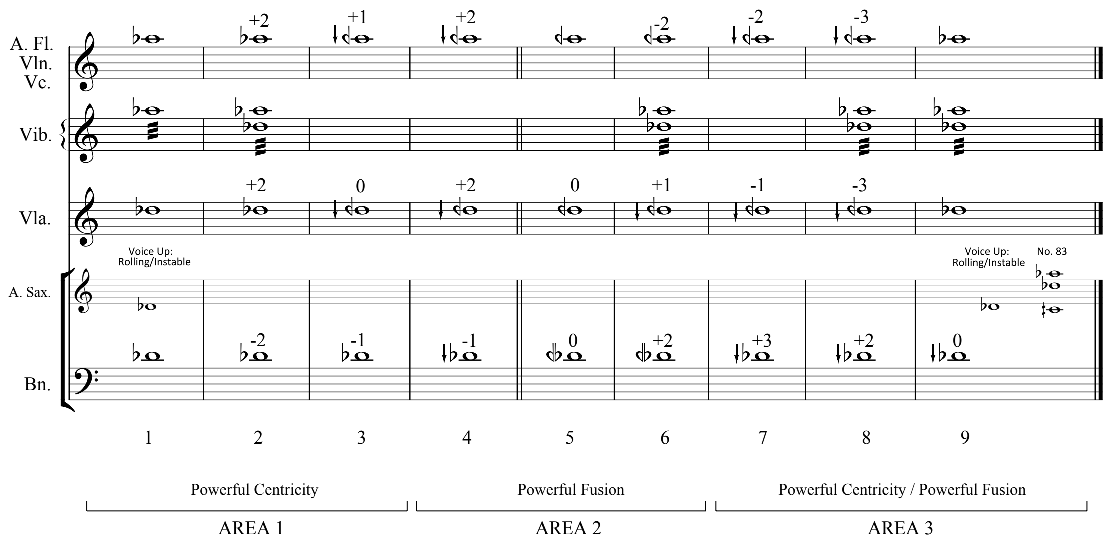

Figure 4.28: Reduction of the centricity-fusion strand across the large-scale form. The harmonic movement of the strand is averaged for each of the nine major sections of *Radial*'s large-scale form. The figure also provides the orchestration of the strand. The three instruments noted on the top staff participate in the centricity-fusion strand in alternation. They do not appear as a doubling or tripling of the line.

Most instruments have only 6 to 12 of these tones—a span which is therefore less than a whole tone—so they certainly don't produce a scale system that relates to, or duplicates at, the octave; rather they create small and fine continuums of pitch and color arranged around one or two notes. Multiphonics themselves exist in such an ambiguous space. It is fair to say the differences between transcriptions of same fingerings remarked upon in Section 3.4 are not simple mistakes on the part of their authors. Rather, they reflect an inherent ambiguity between pitch inflection (harmonic progression) and color inflection (timbral progression) in multiphonics, which produces an interpretative quagmire for their transcriber.[4] Even with the assistance of computers (Veal, 1994), one has to invent a method to filter (interpret) the resultant data down to an approximation of what he or she hears. The accidental system in *Radial* was devised to interlace with that ambiguity and extend its properties. It also has the virtue of a certain flexibility that nonetheless achieves its aim of a delicate continuum of events approaching the JND.

The top system of Figure 4.26 notes that the last sonority of the centricity-fusion strand, multiphonic No. 83, is presented with a Db eighth-tone flat in bass position, rather than a Db three-quarters flat or C quarter sharp. The adjustment is made because the orchestration of the strand tightly coordinates with formants to make powerful harmonics coincide and balance across the group. This ensemble playing the principal pitches of No. 83 produces greater roughness than the saxophone or any other orchestral group in the score; the adjustment matches the tranquil projection of resonance range Db4 by the soloist and the rest of the ensemble. Figure 4.29 details this coordination of fundamentals, harmonics in formants in the centricity-fusion strand, which establishes the acoustic basis of its roughness and propensity for fusion, particularly at the apex of de-tuning in section 5 but also at the flanking portions of sections 4 and 6, and the closing sections 7, 8, and 9 where the outer strands gradually tune-up with the interior (centricity-fusion) strand.

[4] *Radial* makes a formal distinction between timbral progression and harmonic progression. Ultimately the distinction is a matter of degree. Figures 4.20 through 4.22 shows there is no timbral progression without some form harmonic alteration; this applies to alto saxophone timbral space generally, not only the multiphonics of *Radial*. The only deeply problematic stance to take on the issue would be to assume or suggest there is an eighth tone, quartertone or semitone descriptive level for a full body of multiphonics—as opposed to particular substreams—that exists in a flat-timbral space. Unfortunately, short of recordings or detailed descriptive annotations, many multiphonic catalogs fall in that direction.

Figure 4.29: Formant frequencies in the centricity-fusion strand. The bassoon's second harmonic lies in a powerful formant and coincides with the viola fundamental. Formants and strong harmonics for all three instruments coincide at Ab6 (1662 Hz).

Figure 4.29 shows the most important features of the centricity-fusion strand. Clearly, the bassoon's harmonics will double the fundamentals and all harmonics of both upper instruments but if we also consider amplitude, several cases (highlighted in Figure 4.29) are particularly notable. The bassoon 2nd harmonic lies in a powerful formant (giving it more energy than the first harmonic) and coincides with the viola fundamental. The viola's strongest spectral energy usually divides between the first two harmonics; here they are lessened somewhat by energy pouring into the third harmonic, which lies in strongest formant region of the instrument. Consequently, the bassoon 2nd harmonic and viola first harmonic are forcefully balanced. All three instruments in the strand pour strong energy into Ab6. As noted, the viola 3rd harmonic at Ab6 is in its strongest formant. The bassoon 6th harmonic at Ab6 lies in the peak of its upper formant, which gives the bassoon its characteristic nasal voice. The flute does not have formants in the strong sense of these and other instruments, but throughout its register (the lowest fifth excluded) it projects balanced and strong energy across its first two harmonics; it also gives solid energy at Ab6, which competes with the other coinciding spectral partners.

Thus five out of the eight strongest harmonics in the group (two at Dd5 and three at Ab6) coincide with balancing force. As they part they powerfully "de-compose" creating strong beating and high roughness. Fundamentals also beat with elements near their 2nd and 3rd harmonics, so two more of the eight harmonics—the bassoon fundamental and the flute fundamental—come into play. The final member of the eight strongest harmonics—the viola 2nd harmonic—is the most independent of the group. It beats against the coinciding bassoon 4th harmonic, which is only starting to enter its upper formant. However, at the apex of the strand, in section 5, the viola 2nd harmonic rises to D quarter flat 5 and the bassoon *5th harmonic*—clearly in the bassoon upper formant—drops down to F quarter flat 5 above it, and there the limits of their critical bands collide. Figure 4.33, introduced in the following section, charts out the beating and difference tone phenomena of the strand.

4.2.8 Timbral and harmonic grouping through delays, beating patterns, orchestration and ensemble interaction.

Section 2.1 defined multiphonics as sounds with fundamentally ambiguous grouping structures: their interiorities present two or more elements that resist total coupling; but their exteriorities—the large-scale envelope, timbral and spatial cues—insist these elements are one. Such grouping ambiguity is itself a discursive topic in the work, arising from a play between more ornamental and increasingly more structural means of grouping, with the individual multiphonic falling somewhere in the middle of the continuum.

Grouping interaction is an essential experience of the composition; much like the interplay of structural and nonharmonic tones in late Romantic harmony. However, it can be both clear and extraordinarily subtle. A strong surface grouping immediately associates a collection of instruments, perhaps also altering what is heard synthetically and analytically. A weak surface grouping may only influence the nature of local fusion—modulating timbres of things already present. Weak groupings are a matter of structural level, but amplitude is also a factor. A powerfully binding multiphonic exerts force—but in a much more subtle way during its second or third pass through the delay. As in the case of chromatic harmony, non-structural groupings may take a large share of our attention, as we scan their nature and potentiality, but structural groupings have deeper apprehensibility.

Figure 4.30 details surface groupings that articulate and elaborate resonance ranges, the basic building blocks of the work. Later we'll investigate the way these link into larger structural groupings, which complement the harmonic background of the score. At the close of the discussion it will be clear that grouping structure in *Radial* is a unique domain, which like the multiphonic itself, traverses the boundaries of timbral and harmonic projection.

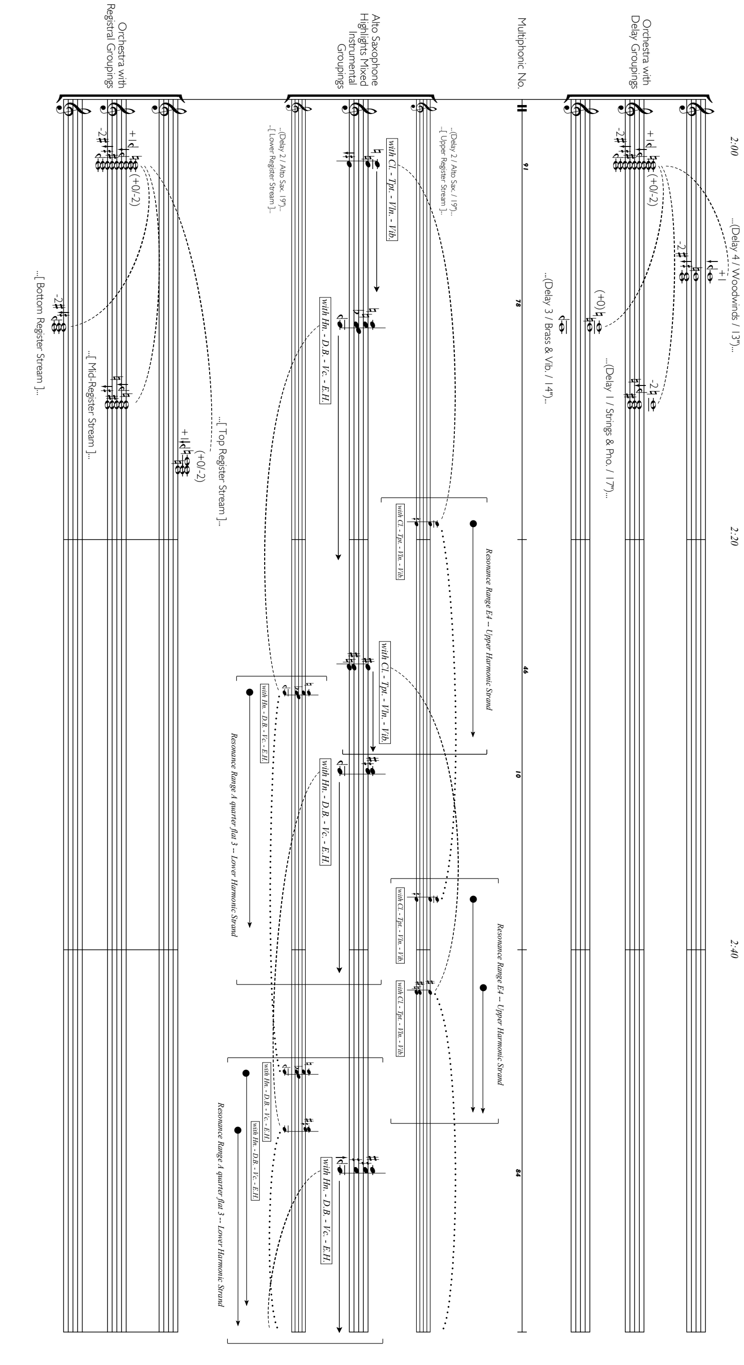

Figure 4.30: Surface grouping structures for the third major section of *Radial*. The figure depicts general registral groups (bottom); orchestral delay groups (top); and the registral streams of the saxophone solo and saxophone delay (middle), which articulate individual resonance ranges.

Figure 4.30 depicts the composite orchestral sonority of time bracket 2:00-2:20—a peculiar set of stacked thirds—in the middle system of the top and bottom brackets. The sonority has a mixed and imbalanced orchestration, so we do not immediately or easily parse it by traditional consorts; it is a “sound-mass event.” (Figures 4.31 and 4.32 analyze its special opacity.) The bottom bracket, taking into account the precise orchestration of chord, shows individual registral groups we might discern by scanning up and down the sound. (Spatial layout in the bottom bracket is for visual convenience only; once articulated our ears are free to explore up and down the sound.) Although other, and often deeper, grouping structures routinely appear upon the surface, registral grouping is an unabated force we may use to interpret the whole.

The upper bracket shows the sound-mass event elaborated by the three orchestral delays. The woodwind group enters at 13 seconds, the brass group at 14 seconds and the string group at 17 seconds. Clearly, the like sonorities and traditional associations of these consorts help the groups to project. In the second and third passes of the delays (not depicted in the figure) group entrances spread out further. None of the newly arising surface entities in the top and bottom brackets project multiphonic structures or characteristics of individual resonance ranges. Instead, they reveal passing perspectives on deeper developments.

The orchestral sound-mass event is arpeggiated by the ensemble and conductor collectively, creating a strong grouping structure of its own. This grouping structure has unique force. It can emphasize its own perspectives, underscore strong or weak grouping structures, and seize upon the serendipitous as well[5]. The articulative salience of the orchestral delay groups is, therefore, modulated by ensemble interaction. Correlate is that figures of the ensemble steadily receive process-oriented variations, built of orchestral condensations then parsed through rhythmic onset dilations, in the orchestral delays.

Strong surface grouping structures arise from the solo saxophone and its delay, depicted together in the middle bracket. The first saxophone multiphonic pierces the sound-mass event projecting both the "high E4—F5—C6" sonority of multiphonic No. 91 and the orchestral group doubling (or nearly doubling) those pitches. That group projects a tightly related harmony, but a fundamentally different sonority. When the saxophone moves away, something *else* is newly revealed to us. (Semiotics would call it "the signified pointing to its index-signifier": like fire pointing to smoke.) Mixed orchestral groups identified by multiphonics are noted in the middle bracket in boxes directly above or below each multiphonic chord.

The saxophone then moves to the bottom of its composite line projecting multiphonic No. 78 and the mixed orchestral group noted below that chord. At the surface, harmonies and mixed ensembles projected by multiphonics are powerfully grouped identities, but they are ornamented and obscured by the groups outlined above and by the sound-mass itself, a force from which every group—structural or otherwise—must first stand out. Thus, strong grouping structures stand out, but new forces continually arise.

[5] Seating arrangements encourage consort interaction and rehearsals usually isolate individual strands with the soloist before putting them together. Thus various collectives are well established.

Just before the onset of time bracket 2:20-2:40, the saxophone delay (the last of the four delays) enters at 19 seconds. The grouping forces of the initial saxophone line recur in the delay, now with associational (motivic) weight, and are further developed by the soloist. Brackets show the saxophone solo and saxophone delay filling out the wider resonance ranges of the upper register stream (the upper harmonic strand) and the lower register stream (the lower harmonic strand). The process compounds in time bracket 2:40-3:00. Dashed ties moving from the solo into the delay streams in Figure 4.30 indicate a repeated event with potent amplitude. Dotted ties are used for second delay appearances, indicating the event is audible but with significantly less force.

The semiotic distinction between saxophone multiphonics and mixed instrumental groups is emphasized because “saxophone meets saxophone in the delay” produces something fundamentally different. The qualitative alignment of interior (harmonic) and exterior (timbral) elements between multiphonics from the same resonance range creates de-composition into a larger, definite identity (by comparison think “fire on fire.”) Thus two coherent sound-masses, a higher body and a lower body, gradually come into view.

Because these bodies have increasing overlap across the work, the mixed orchestral groups they project both elaborate *and distinguish* the merging strands. The middle bracket of Figure 4.30 shows mixed orchestral groups stay attached to their respective streams as they develop. Orchestration gives similar grouping coherence to the ascending and descending harmonic strands in each of the nine major sections (This point is illustrated in greater detail through Figure 4.34 at the end of the chapter.)

Ultimately, the “semiotic” difference between multiphonic and mixed orchestral group is acoustic. Mixed orchestral groups of the outer-strands have spectra different from the multiphonics they are associated with. (They have correspondences, but they are not direct or “nearest” spectral reconstructions.) They also don’t have the uniform de-compositional weighting of the centricity-fusion strand. They are organized (where possible) so that harmonics do not coincidence, strong formants spread across the sonority, and particular elements under-balance. This produces a denser sound than two corresponding multiphonics (one for each outer strand), which still has an undulating, ambiguous relation between pitch and timbre, much like multiphonics themselves.

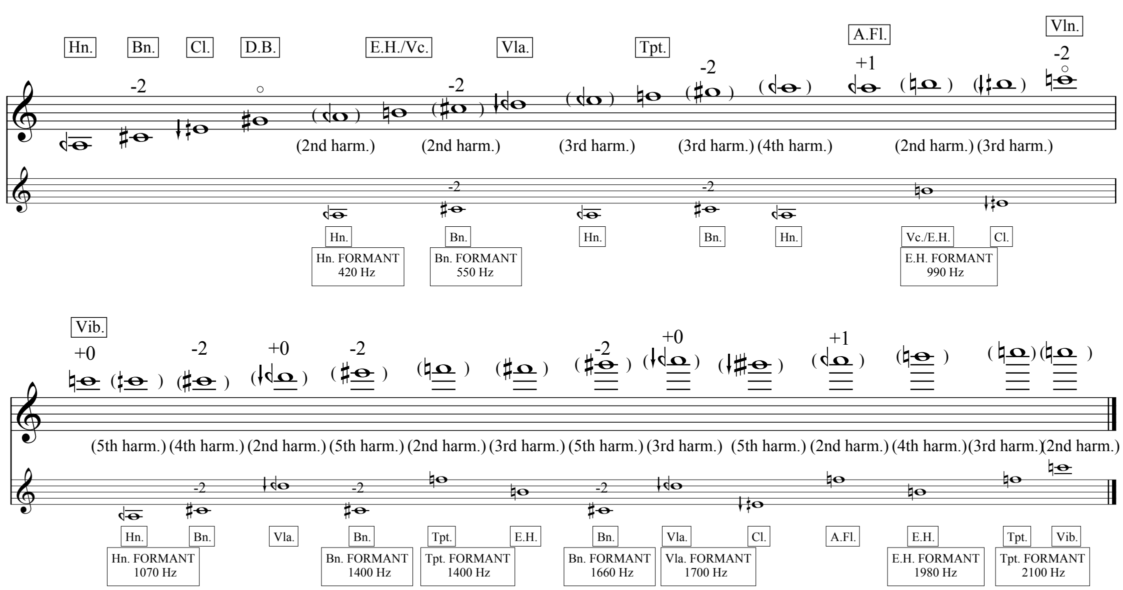

Figure 4.31: Spectral opacity in the first orchestral event of Figure 4.30. Fundamental tones in the sonority are listed by instrumentation above the staff. Harmonics in the primary staff are shown in parentheses; fundamentals in the lower staff demonstrate their origins.

Figures 4.31 and 4.32 demonstrate these features in the first sound-mass event of Figure 4.30. Unusual harmony, choices of orchestration, and fine accidental de-tunings achieve the non-coincidence of harmonics. As stacked—but diverse—thirds, individual fundamentals form a contiguous string of critical bands. Of the twenty most pronounced harmonics (pictured in Figure 4.31) only two coincide—the top two C7s, from the trumpet 3rd harmonic and the vibraphone "2nd harmonic" respectively.[6] Fundamentals and upper harmonics viewed collectively show that all critical bands beyond the bottom three intervals meet with increasing division by roughness producing partners. These factors establish the opaque spectral density of the whole. (Fundamentals spread by "thirds," that gradually contract, are common throughout sections 1-4. In sections 5-9 fundamentals generally form tenser "seconds" that contract towards unison.)

The spreading of formants is shown in Figure 4.31. Abutting formants appear (as discussed earlier) in the instruments of the centricity-fusion strand, but rich formants of the outer strands (horn, English horn, trumpet) are widely displaced. Harmonics in these formants variously balance and over-balance neighboring sounds. The horn and trumpet 2nd harmonics over-balance sounds found within a half-step on either side of these pitches, and there are no frequencies within a half-step on either side of the English horn 4th harmonic. More evenly balanced formants are the English horn 2nd harmonic and powerful clarinet 3rd harmonic; the horn 5th harmonic and vibraphone fundamental; and rich, though not "balanced," is the high trumpet 3rd harmonic and vibraphone "2nd" harmonic. Accordingly,

[6] The actual vibraphone 2nd harmonic is not in this position, but higher harmonics are sculpted into a 2:1 ratio with the fundamental. For further detail see Section 2.3.1.

some instruments easily pierce the texture, while others de-compose or must instigate powerful amplitude peaks to rise to the surface. This strengthens the undulating character of the sonority and draws an acoustic analogy to the sound of the alto saxophone multiphonic. In the most stable multiphonics [B/Instb 0-1], the manipulation of upstream resonances can make each of the three basic tones over-balance the other two sounds. In *Radial* the soloist projects multiphonics both as vertical harmonies and “unfolding” sounds in the manner described. A final critical factor in establishing the characteristic undulation of the greater sound-mass is the use of conventional timbres in the orchestra, through which familiar sounds regularly emerge from the whole. The surface of *Radial* is complex, but by avoiding multiphonics, mutes and other special effects in the orchestra, the “deck is not stacked” against the listener. This encourages a more participatory listening—shifting freely between the analytic and synthetic—wherein listeners can pursue the origins of particular sounds and groups and contemplate the forces driving the totality of the work.

Figure 4.31 is illuminating, but clearly we do not hear each of these notes; rather we perceive a mixture of pitches, beats and roughness. Figure 4.32 interprets the sonority from this perspective. It shows only the fundamentals of the sonority and prominent difference tones created by interactions of fundamentals and upper harmonics alike. What emerges is that audible beating in the sound complex (from 0.75 Hz to 14 Hz) derives from all three strands, but that roughness (20 Hz to 66 Hz) is largely dependent on the presence of the centricity-fusion strand. The solid, contiguous strand of critical bands formed by the fundamentals is also dependent on the centricity-fusion strand.

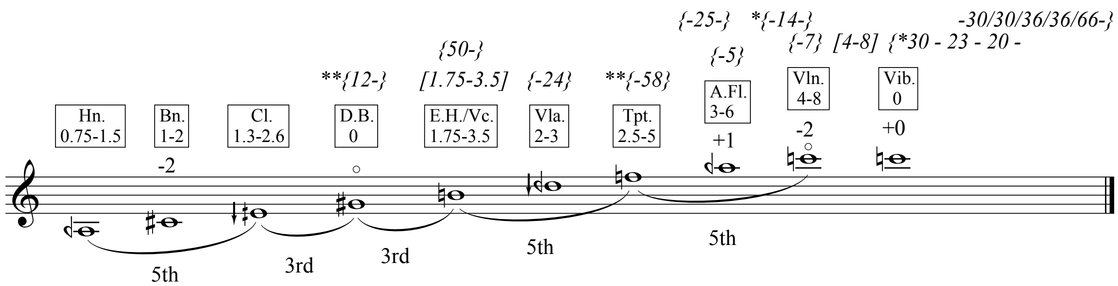

Figure 4.32: Fundamentals and prominent difference tones in the first orchestral event of Figure 4.30. Fundamentals are listed by instrumentation. The boxes also show the beats created by each instrument and its voice in the delay. Straight brackets show beats between close fundamentals, and curled brackets show difference tones (beats and roughness) involving harmonics.

The slurs in Figure 4.32 connect fundamentals linked to the ascending and descending harmonic strands. Without the centricity-fusion strand, the full sonority (in sections 1-4) would have substantial gaps. Boxes containing instrument names show beating patterns produced by each instrument and its voice in the delay. This range of activity is 0.75 Hz to 8 Hz. Low frequency beating between close fundamentals is shown in straight brackets: the English horn and cello, and violin and vibraphone produce such beating. The range of activity is 1.75 Hz to 8 Hz. Curled brackets show difference tones between fundamentals and an upper harmonics (single dashes point to the direction of the harmonic) and between two upper harmonics (these are represented by numbers with dashes on both sides, the harmonics themselves are not shown.) Curled brackets show a range of activity from 5 Hz to 66 Hz. Asterisks point to difference tones above 8 Hz (the beating limit of the delays) produced by the outer strands alone. (The single asterisk points to balanced pairs, the double asterisk points to unbalanced pairs.) Therefore, the outer strands produce significant beating from 0.75 Hz to 8 Hz, but beyond this they produce only balanced difference tones at 14 Hz (between the English horn 2nd harmonic and powerful clarinet 3rd harmonic) and 30 Hz (between the horn 5th harmonic—in a formant—and the vibraphone fundamental), and weak difference tones at 12 Hz and 58 Hz. Most of the higher frequency “roughness-producing” beating patterns—20, 23, 24, 25, 30, 30, 36, 36, 50 and 66 Hz—come from the centricity-fusion strand or its interactions with the outer strands. (Each of the ten cases listed above are balanced or nearly balanced.)[7]

Figure 4.33 shows that beating patterns produce a large-scale shape of their own across the score of *Radial.* The figure is a “beating key” for the whole of the composition. The bottom five systems show the registral range (Ab3 to Db6) of fundamentals at the start

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[7] It might be noted that some of these difference tones—those from 20 Hz to 66 Hz, but especially the higher ones—could be heard as low frequencies. However, difference tones heard as independent low frequencies, created between two separate instruments, usually need to be formed between two fundamentals. In this way the difference tones of the higher harmonics from both pitches also reinforce perception of the new low frequency sound. The cases here largely produce roughness because they are between odder pairings—for instance, a fundamental of one instrument and a fifth harmonic of another. Such a pair would have to produce the audible frequency alone. (It is unlikely that the difference of the 2nd harmonic from the first instrument and the 10th harmonic of the second instrument would contribute significant support.) Spectral analysis confirms low frequency difference tones occur in *Radial*—there are usually 3 to 7 distinct lines—but audible tones are typically fleeting and quite soft. There are exceptions. *Radial* regularly produces difference tones in the region 180 to 190 Hz. At the start of section 5 a difference tone in the 90 Hz region is created between trombone and viola fundamentals, which corresponds to roughness in the 90 Hz region and 180 to 190 Hz regions created by the centricity-fusion at its apex, producing a powerful effect. Presumably, combination tones also play a role by highlighting audible high harmonics, like those depicted in Figure 4.33.

of the score. The first measure shows the beating effect of the “+1” interval in each of these positions. The second measure shows the same for the “+2” interval, followed similar data for eighth-tones, quartertones and semitones. Thus, fundamentals at the start of the score—

Figure 4.33: “Beating key” for the centricity-fusion strand and *Radial*’s large-scale form. Each measure shows the beats produced by a specific interval at six different frequency ranges. The top bracket synopsizes what is active in the first half of the composition and the second half of the composition.

depending on the size of surrounding adjacencies—produce beats from 0.75 Hz to 64 Hz. The sub-bracket (enclosing Db4 to Ab5) shows the range of fundamentals at the close of the score, which produces beats from 1 Hz to 48 Hz. In the last sections of the work adjacencies tend to be smaller than the semitone, so the window of beating activity decreases still further to 1 Hz to 24 Hz. This narrowing of acoustic energy into the domain of audible beating (frequencies below 20 Hz) explains the more intense ringing of the second half of the composition, which reinforces the large-scale division of the composition into two parts.

Figure 4.33 also provides a clear picture of beating development in the large-scale unfolding of the centricity-fusion strand. Throughout sections 1-4 the bottom voice of the strand falls one quartertone and the top two voices raise one quartertone (see Figure 4.28), creating harmonic adjacencies spread by one semitone. Thus over sections 1-4 the centricity-fusion strand gradual moves from beating patterns shown at the left of Figure 4.33 to beating patterns shown at the right of Figure 4.33. The apex of beating is reached at the start of section 5 and the process then reverses, with beating activity gradually scanning back to the left of Figure 4.33 throughout sections 5-9. (As mentioned before, an additional layer of beating and roughness forms and relaxes across sections 7-9 as the ascending and descending strands gradually merge with the centricity-fusion strand.) Thus, the centricity-fusion strand creates a beating to roughness development much like the flow of tonic and dominant harmony in a classical binary form composition: "audible beating & low roughness → high roughness | high roughness → audible beating & low roughness." Clearly this confirms the larger harmonic division of the work into two broad halves; but more importantly the rise and fall of roughness creates a spacious fusion-wave that modulates our perception of steadily unfolding surface grouping structures.

Figure 4.34 demonstrates that the orchestrations of the mixed orchestral groups projected by the saxophone solo also form large-scale grouping patterns. Generally, the descending harmonic strand is given a brighter orchestration and the ascending harmonic strand is given a darker orchestration. The distinction is strongly pronounced in section 1 and more muted in section 2. This creates an A | B pairing of orchestrations that continues throughout the work. In section 3 the distinction becomes pronounced again, but through new orchestrations that establish an A2 group; it is answered in section 4 by a near exact repetition of the B group. Thus orchestrations of sections 1-4 create a larger ([A1 | B][A2 | B]) unit. This larger group is repeated in sections 5-8 with a reprise of the "A" group in section 9. All of this is laid out, together with the actual orchestrations of each group, in Figure 4.34.

The total grouping effect of the orchestration is more robust than a simple binary division of the form. It creates a hierarchical structure that strengthens the four time span reduction levels identified in Figures 4.22 and 4.23. To clarify this point Figure 4.35 re-presents Figure 4.22 adding the corresponding orchestral grouping structures to its four time span reduction levels and broader branching structures.

In *Music as Social Life* Turino (2008) forms a distinction—or graduated continuum—between participatory musics and presentational musics, noting that cyclical structures are common features in art forms that call for greater degrees of participation from performers and audience members. The cyclic even temporal boundaries of *Radial*'s

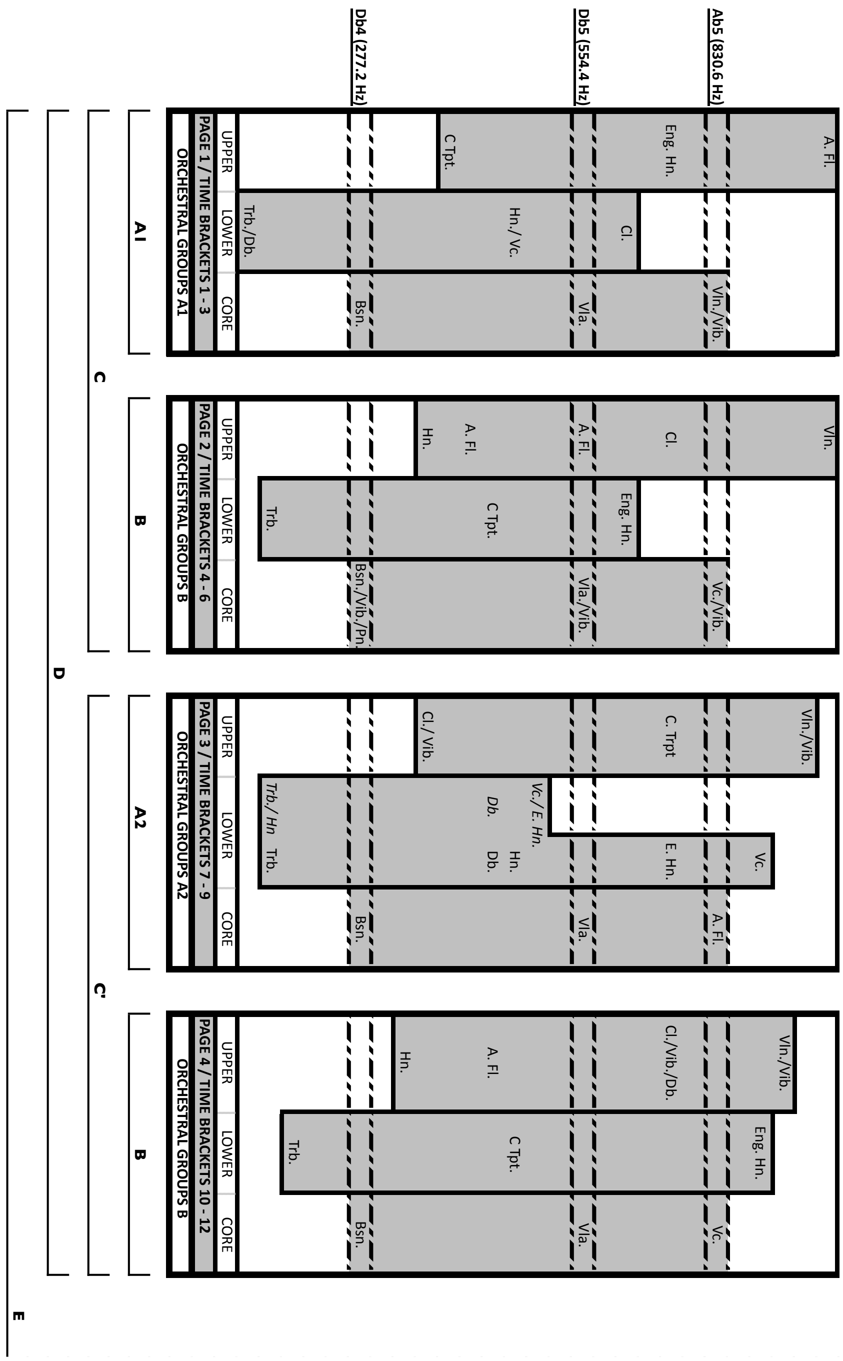

Figure 4.34: Large-scale grouping structures of the mixed orchestral groups. The upper strand has brighter timbres and lower strand darker timbres. This distinction is stronger in the A sections and weaker in the B sections. Contrasts at the level of the A and B groups form the larger grouping structures approaching level E.

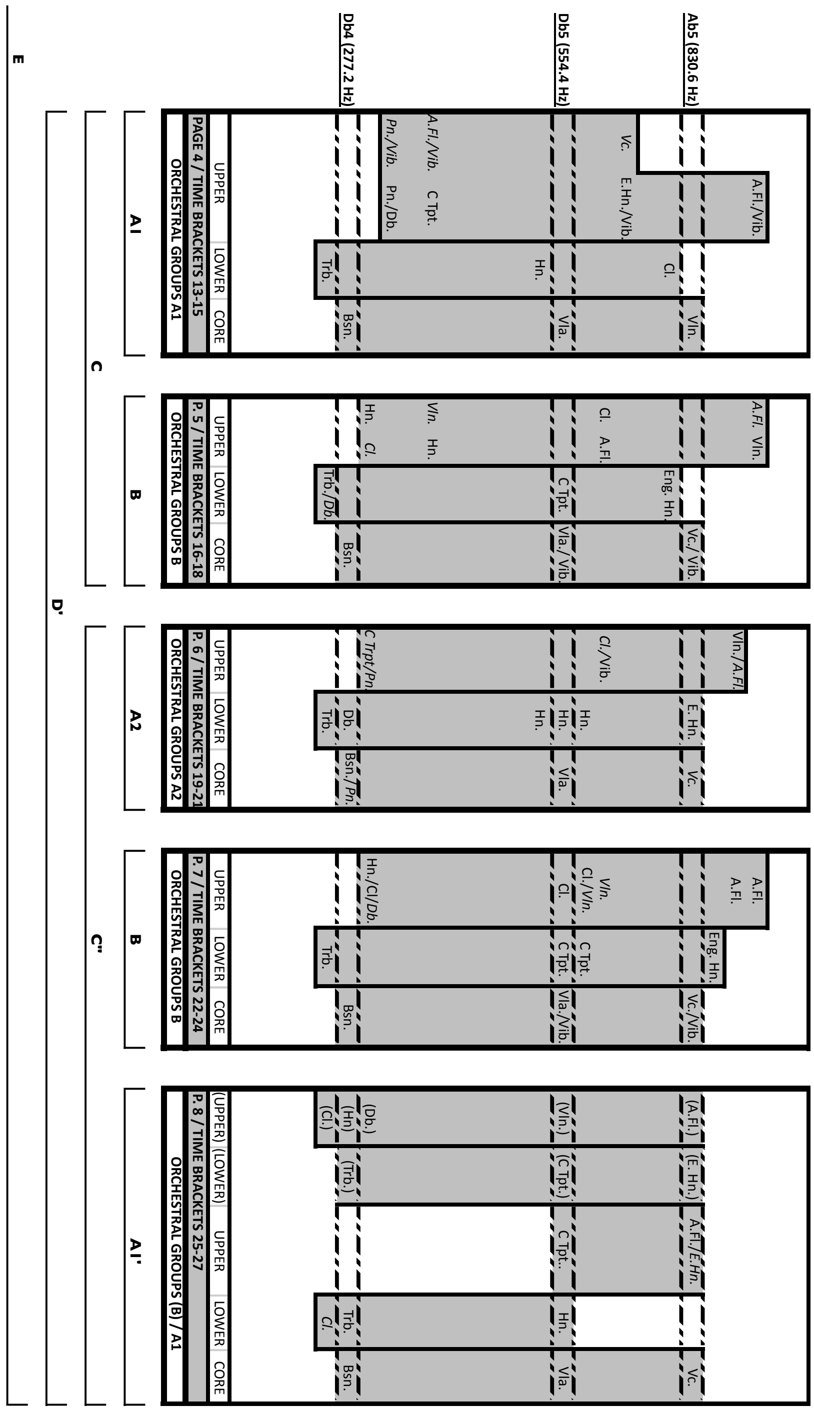

Figure 4.34 (Continued)

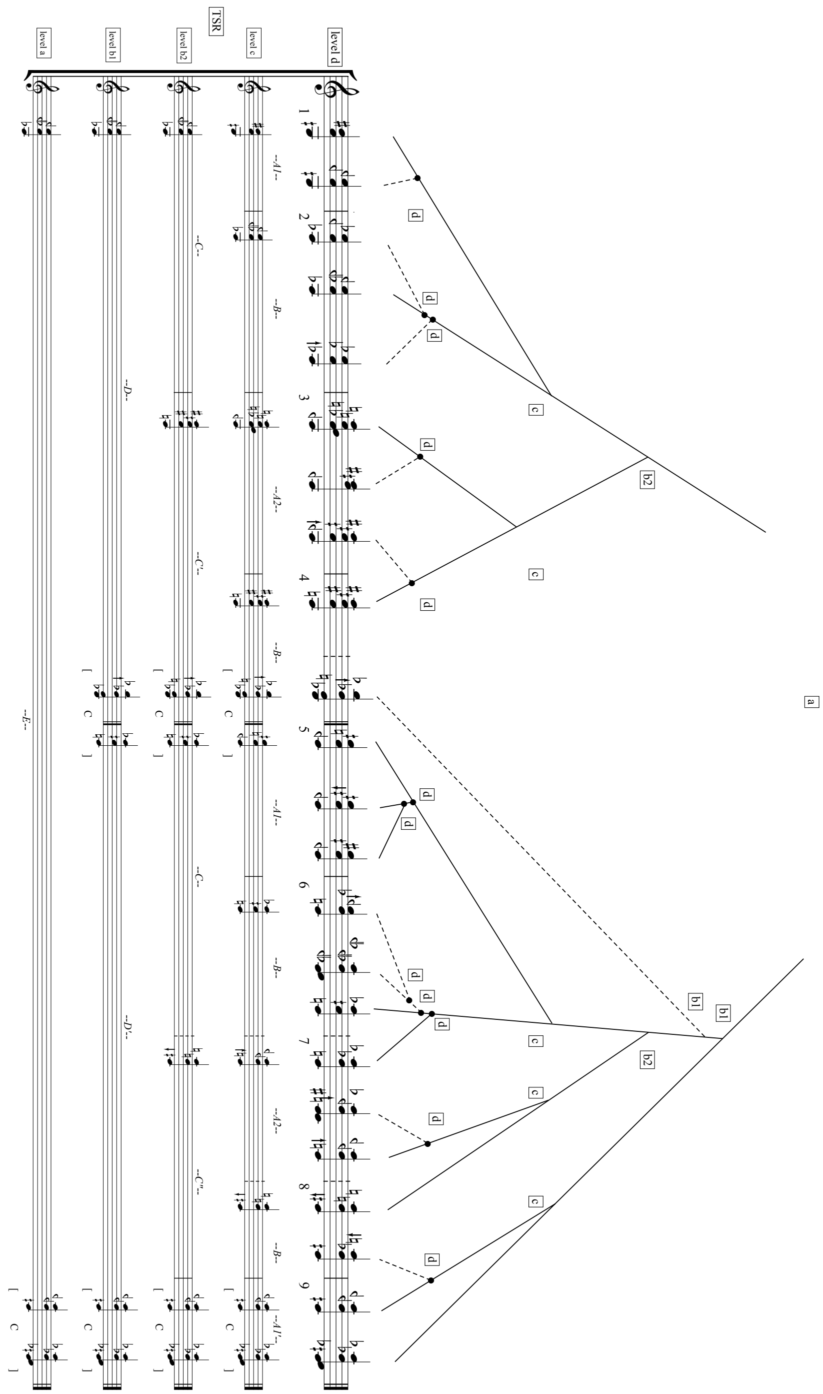

Figure 4.35: Orchestral grouping structures annotating the time span reduction levels of Figure 4.22. The TSR levels and complete branching structure for the ascending harmonic strand are shown with the orchestral grouping structure.

nine large sections, the steadily unfolding surface grouping structures, and the large-scale grouping cycles that subsume one another, are intended to embrace and sustain active participation—from performers and listeners—in exploring the sonorities, spaces and networks embodied in *Radial*'s design. The same goal is also pursued through clear perspectival divisions of the work—the simultaneous layers of the three harmonic strands and the two timbral domains of soloist and orchestra. Scelsi's masterpiece *Anahit* is a precisely chiseled portrait made from a recording of one of his many ecstatic electric organ improvisations. By contrast, we could say that *Radial*, a work inspired by Scelsi's masterpiece, aspires to be an ecstatic—near alchemical—structure for participatory action, which through subsequent performances will create a long narrative about and between the space and the actors.

5 Conclusion

The preceding chapters have provided a new general definition of multiphonics identifying five broad categories of multiphonic sounds, surveyed the musical and scientific literature on multiphonics and related acoustic concepts, formulated an idiomatic multiphonic case study "alto saxophone multiphonic space," and presented an analysis of that space at work in the alto saxophone concerto *Radial*. Appendices A and B have given readers a sociological and historical account of multiphonics and additional discussions of several key terms and concepts. What has been achieved is indebted to a new body of research in multiphonics, acoustics and music cognition that has been steadily growing from the 1950s onwards. Thus this work is a synthesis of efforts in different disciplines, but at the same time significant new material has been put forward, which merits further elaboration and points to new topics of investigation.

Each of the five multiphonic categories—harmonic series-based, collateral, radiated, multi-driver, and non-coincident multiphonics—can be explored in greater detail. For instance, it would be useful to identify all harmonic series-based multiphonics for each of the major woodwinds, including common variants of particular instruments such Boehm and Oehler clarinets and German and French bassoons. The material would form an excellent basis to investigate systematically the collateral multiphonics of these instruments.

Brass multiphonics can be studied with much greater precision by observing the contrasting and potentially overlapping categories of harmonic series-based multiphonics, radiated multiphonics and multi-driver multiphonics. Short of such basic distinctions, it has perhaps been difficult to describe brass multiphonics, which have no single comprehensive acoustic reckoning or catalog. Valuable work has been done (as noted in Chapter 3), but most of these investigations briefly introduce basic theory appropriate to collateral multiphonics (which are largely non-idiomatic to brass instruments) and quickly move to literature surveys. With more appropriate categories identified, new work can begin.

The instrument-specific case study “alto saxophone multiphonic space” is a powerful tool that can readily be used to create similar spaces for the other orchestral woodwinds and members of the saxophone family. These would usefully supplement extant multiphonics catalogs and likely inform the development of future compendiums. On the side of theory and research, a full collection of such spaces would help define woodwind multiphonics generally, and likely reveal further properties shared amongst conical instruments and shared amongst cylindrical instruments that have not been previously identified or properly appreciated. The collateral resonance zone of the saxophone, which relates more generally to unique acoustic behavior in the area of the second harmonic on all woodwinds, deserves specific scientific scrutiny.

The “alto saxophone multiphonic space” model can be further developed. A study on correspondences between resonance ranges and the character and permutational limits of fingerings would be illuminating. For instance, resonance ranges in the present work have been formulated for each quartertone, wherein eighth-tone variants are systematically grouped to lower quartertones. However, keyhole anatomy might suggest more subtle groupings—up in some cases, down in others—that would give greater structural coherence to individual resonance ranges. The spatial representation of the timbral space can be improved and likely extended to three if not more dimensions. An interlay of scalar level *e* and timbral space footprints for each note could perhaps be finessed to create a "regional" timbral space representation. (On a more tangential note, such spatial representations of distortion continuums could suggest new analytical tools for the study of individual spectral compositions.)

Work in the present document was done by hand and by ear—placing spectra, transcriptions and fingerings in electronic databases would assist further and more rapid progress. Such a model called "conTimbre" is under development by Thomas Hummel and a large group of collaborators, but it is presently a query-based tool (Hummel et al., 2006). The multiphonic categories of Chapter 2 and "alto saxophone multiphonic space" of Chapter 4 have broad acoustic underpinnings that reveal deep and essential trends one would surely want to see before devising more specific queries. Ideally a database (instrument specific or more broadly conceived) would reveal further trends that gradually enrich the models presented here, while also servicing more idiosyncratic composer or composition-specific concerns.

The present work has prospects for guiding stronger interventions and charting new directions. The proposed study on correspondences between resonance ranges and fingering mechanics could suggest interventions in modern keywork, creating greater transpositional variety similar to the Kingma flute (though within the limits or perspective of conical bore structure). It is intriguing that the flute, now the cylindrical instrument par excellence, has in the past had more conical designs. Perhaps an investigation (or re-investigation) of multiphonics on baroque instruments would reveal fascinating but forgotten materials and hybrids.

The physical circumstance of non-coincident multiphonics has been presented as the smallest or most limited of the five broad multiphonic categories, but it may be the most provocative category for intervention and exploration. By looking at the map of a multiphonic space one could choose strategic frequencies to drive into the bore of an instrument through simple piezo drivers. For instance, how would the basic lexicon and structures of "alto saxophone multiphonic space" be modulated by driving the frequency of a prominent resonance void—such as B3 or B quarter flat 4—directly into the instrument?

Collateral multiphonics and non-coincident multiphonics have clear spatial implications, which deserve investigation. Suffice it to say that the basic harmonic and amplitude ambiguity of a collateral multiphonic that leads to the question "which of these is the central pitch?" also leads to the question "which point of projection is the principal radiator or the sound?" Ultimately, the harmony and microtonality of collateral multiphonics is spatialized (the same is evidently true for non-coincident multiphonics.) In response to this fact, Taimur Sullivan has recorded the multiphonics of the principal resonance ranges traversed in Radial in a unique arrangement with five close-range microphones. The surprising results of this study are forthcoming.

Given the rich spatialized radiation of collateral multiphonics and the status of *Radial* as a work responding to the combination tone and standing wave compositions of Alvin Lucier, it is hardly surprising that *Radial* itself has large spatial implication and sits at the divide of installation and concert-work. The composition is captured in recording, but hearing it live is a different, more immersive experience because the detail we rely on to pierce its surface delicately reconfigures as the listener turns her or his head. The unique crossroads of *Radial* subsequently led to the three-part cycle *Bazo—Diabazo—Allo Akousma*, which combines a forty-speaker installation, an instrumental composition, and a conceptual work exploring four-dimensional recordings and auditory anatomy.

It is hard to imagine what other directions this focused but deeply multidisciplinary study might inspire. In one recent case, the notion of spatialized microtonality interested the emerging composer Tristan Perich informing the development of his “Microtonal Wall,” which explores spatialization within a provocatively fine grid-based conceptual design. Further reactions, piecemeal or unsuspected, would be a great reward. The present work seized upon multiphonics to establish an acoustic basis for exploring concepts in American experimental music with asymmetrical and hierarchical structures that relate to the materiality of sound and the foundations of perception and cognition. It will be a vindication if its implications—large or small—strengthen the alliance of social, scientific and musical inquiry that has long empowered artistic expression.

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Appendix A: Global and Historical Perspectives on Multiphonics

A.1 Perspectives on a Global History

When a monophonic instrument – like a flute or a clarinet – produces two or more notes at once, we call that sound a multiphonic. Multiphonics produced by flutes or clarinets are familiar examples, but many instruments and various performing practices can produce multiphonics. It is often said that multiphonics first appear in the late 1950s in the work of postwar European composers, from where they quickly find their way into jazz and gain wider public visibility. This story, however, is mistaken in nearly every respect. In this chapter we will stress that multiphonics appear in music traditions around the world, and that their history is no easier to trace than the history of music itself. We'll then review their early history in classical music and jazz, and look more closely at their remarkable rise in the late 1950s and beyond.

Documentation of the famed multiphonic Australian didjeridu begins 1000 years ago, but Aboriginal people place its birth in "Dreamtime" – or Aboriginal prehistory (Knopoff 2010; Marett et al. 2010). The Slovak fujara and Scandinavian willow flutes are, from an acoustic perspective, close cousins of the didjeridu but they developed independently in Europe and Russia. Though their antiquity is evident in many archaeological findings, no particular excavation points to a founding practice (Oramo and Kolehamainen 2010; Rybaric et al. 2010).

The throat singing traditions of Central and East Asia rely on multiphonics (Pegg 2010), as do some Inuit (North America) and Xhosa (South Africa) vocal genres (Nettl et al. 2010; Rycroft et al. 2010). The beginnings of these traditions certainly escape the written record; and being vocal genres they're unlikely to produce archaeological findings. However, the Inuit of North America, the Ainu of the former Japanese territory of Sakhalin (annexed by the Soviet Union in 1945) and the Chuckchi of Russian Siberia do share a particular style of vocal multiphonics that is conspicuously united by additional formal markings, such as the face-to-face pairing of female vocal performers. Drawing on original fieldwork in each setting, Nattiez (1999) argues these features must have been in place 4000 to 5000 years ago when the groups first separated from an earlier common culture. The emerging linguistic and genealogical data Nattiez cited to bolster his view has since become established (Comrie 2009) adding considerable force to his case. Singing styles may lack primary historical artifacts, but in this case we are hugely fortunate to assert a particular mode of vocal multiphonic production likely in use 4000 to 5000 years ago.

The ubiquitous mouth harp doesn't require vibrating vocal cords, but it is comparable to throat singing practices. Through its overt use of the vocal tract it produces similar harmonic melodies and harmonic clusters, but these are developed from and accompanied by the drone of the instrument's free reed (instead of the performer's vocal cords). Until recently the mouth harp was thought to have indigenous traditions in Europe, Asia and Oceania (Fox 1988; Wright 2010). However, Dargie (2008) now adds an indigenous Namibian tradition (Southern Africa) to the list, significantly complicating established theories of a late and centralized origin. Probably the powerfully documented spread of metallic heteroglot (horseshoe-shaped) mouth harps – which were popular trade route items in Europe and Asia since the late Roman era — has distorted historical discussions. The historical record of metallic heteroglot mouth harp only became clearer still when European makes were widely distributed during the Age of Exploration (15th-17th century). This unambiguous transmission is usually used to explain the Inuit mouth harp tradition. However, it is worth stressing that the Inuit tradition also includes an idioglot (lamellate) mouth harp about which we know comparatively very little – except that it is precisely like those used in Siberia, Southern Asia, Oceania and, it now seems, in Southern Africa. While avoiding a deconstruction of the mouth harp's history here, it is well to note that the origins of the earlier and widespread idioglot variety remain, at the very least, much more difficult to reveal.

Scholars continue to argue over an enigmatic Upper Paleolithic painting at Trois-Frères Cave in Ariège, France. Rault (2002) sides with the original identification of Henri Breuil (1952), who notes a man dressed as a bison playing a musical bow resonated at the mouth. The painting is dated to 13,000 BC. For Breuil the picture depicts a shamanistic rite associated with the hunt, which is at once an appeal for strength and an act of contrition. If Breuil and Rault are correct, then the instrument would have created multiphonics in much the same way as the modern mouth harp.

Disputes over this interpretation are related to a more general argument as to whether the bow itself was originally an instrument of music or a tool for hunting. In the

Eastern and Western hemispheres, amongst peoples who are presumed to have split some 15,000 to 20,000 years ago, it is used indigenously for the hunt and as a musical instrument – with mouth resonated traditions known throughout Africa, Oceania and the Americas. The bow's Gordian historical record does little to resolve questions about its origins and first applications (Balfour 1899; Rycroft 2009).[1] But that controversy lies beyond our concerns. More pertinently Nattiez (1999) observes that dispersed social and technological correspondences are ultimately given explanations which are either: 1) diffusionist; 2) phylogenetic; or 3) universalist. For this brief sampling of global multiphonic practices, it is sufficient to indicate the mouth resonated musical bow either has an extremely ancient phylogenetic precursor (which spread slowly and widely), or a general universalist tendency (that is, it draws on features common enough to be invented by peoples again and again.) The evidence clearly does not suggest a diffusionist dispersal (meaning the mouth resonated musical bow was a quirky discovery that managed to move fast and far through one of several peaks of rapid social interchange in the last 1000 years.)

[1] Upon close examination the situation is enormously complex. Numerous peoples, such as the Khoisan of Southern Africa, use one instrument for both hunting and music. But others, such as the Dan of Côte d'Ivoire, maintain a separate device for each purpose. Still others possess only a hunting bow or a musical bow, but not both. For instance, the Zulus of Southern Africa knew only the hunting bow until recent times. Thus, it is not simply a question of which application came first: some scholars argue the devices may have entirely separate origins, but that in particular contexts they merged into a single tool. No origins story has yet gained wide acceptance, but mythology reminds us the two instruments (or two functions) are often deeply intertwined: the Ancient Greek god Apollo and the Hindu god Shiva both claim dominion over archery and music, while the Japanese god Ameno Kamato is said to have created the first koto by placing six archers' longbows together in tight formation (Rycroft 2009).

The gourd resonated musical bows of Equatorial and Southern Africa achieve multiphonic effects without recourse to the vocal tract. These instruments came to South America through the Atlantic slave trade. In Brazil they are called berimbaus; documentation there begins in the early 1800s. While issues of style and terminology confirm the transmission (Béhague 2009), it is has long been unclear if the process can account for the gourd resonated musical bows of Central America or of the Apache still further north (Nettl et al. 2010; Rycroft 2009; Saville 1898).

Interestingly, here as elsewhere, it seems the timbral curiosities of indigenous North American music are often underplayed: witness the "throat singing of Asia," "mouth harp of Eurasia" and "musical bow of Africa." This view results, in part, from tempering North American examples to match a conservative depiction of indigenous South American traditions that took shape gradually from the Spanish and Portuguese colonial era, into the middle of the twentieth century. However, recent archaeological discoveries (Olsen 1986; Robertson ed. 1992) from Pre-Incan and Pre-Aztec societies are helping jolt that pattern of "selective" description and underestimation.

Specifically, remarkable clay instruments, some as old as 3000 years, have come to light from sites throughout the western perimeter of South America and stretching into Central America as far north as Veracruz. Amongst the findings are complex inharmonic whistles that produce multiphonics by coupling two or more resonating chambers in the body of a single aerophone. The findings also include multiphones (like the Ancient Greek aulos), which lead a single mouthpiece into separate resonating bodies—some with their own finger holes, but each with their own tubular ending. The early Aztecs designed these flutes with as many as four separate tubular endings. Such extreme cases speak to the difficulties of categorically separating multiphones and inharmonic whistles. (Clearly a four-bodied multiphone is meant to be played and heard, in at least some respects, as a single sounding device and not as four separate instruments.) A third species, from excavations in Chile, illustrates the same point: these flutes flare into two separate tubes in the middle, but then rejoin to produce what is essentially a single composite sound replete with harsh low frequency beating.[2] Still more confounding are instruments found in sites scattered across Mexico, Ecuador and Peru that possess two blowholes and multiple interconnected resonating chambers (Rawcliffe 1992). Summarizing these findings Robertson (2009) notes the diverse and early clay instruments of South and Central America prefer tonal and timbral organization over octave correspondence and registral range; and that their animistic shapes suggest a goal of creating distinct voices for the gods in ritual contexts – a line of argument firmly echoing Rault's (2002) and Breuil's (1952) interpretation of the musical bow scene painted at Trois-Frères Cave.

These remarkable instruments fit well with a new ethnomusicological portrait of

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[2] Pictorial evidence suggests these harsh-voiced flutes may have been used in consorts to create a noisy hocket-based music. (Though its antiquity in South America is still much debated, the hocket is a familiar feature of many indigenous traditions across the Andes and greater South America. It is not limited, as is often assumed, to the South American traditions of the Afro-Hispanic or Afro-American diaspora.) Interestingly, these unique clay instruments also have a modern voice. The Araucanian Indians (Chile) fashion a similar device by binding together two stalks of bamboo and tuning them less than a major second apart through the addition of water. Ultimately, we do not know whether the cane or clay technology is older. The impressive antiquity of the clay species obtains firstly from its material durability (Robertson and Béhague 2009).

South America that began coalescing in the late 1960s (Boilés 1967; McCosker 1976; O'Brien 1976; Hill 1979; Robertson 1979; Seeger 1979 & 1987). Today scholars (Robertson, ed. 1992) are quick to note that well into the 1950s, a selective western-oriented musicology allowed sudden diffusionist changes in post-Columbian South America (originating from both internal and external forces) to confound our perception of the continent's earlier — and in many cases, still present — indigenous condition, thereby creating an idealized tradition suffused with "proto-European" features. This portrait of an early musical culture largely concerned with simple, near modern scales and clean tone production (Robertson 2009) is a particular concern for any history of multiphonics because the image is used erroneously, and inadvertently, to circumscribe the temporal and geographic scope of unusual or "exotic" sounds in Western and non-Western world traditions alike.

Rarely does a revision in music history or music theory seek to venture into the Middle Paleolithic (for one volume with examples see Wallin 2000). But Robertson is bold in dismantling this image of the Americas as a musical outsider. After reflecting on the richness of fuller, more contemporary archaeological and anthropological record of North and South America she concludes "we can also assume that musical sounds and instruments were brought across the Bering Strait from Asia in migrations which began 40,000 to 70,000 years ago" (Robertson 2009). Perhaps she is referring to nothing more than a sensibility, a vocal tradition and an instrument or two. In any case, she is laying to rest an old conceptual engine, which relied on the example of a simple indigenous music in the Western hemisphere to hedge descriptions of non-Western features in music traditions around the world. For the present study one implication is clear: the Americas are no multiphonic outlier.

A.1.1 Towards a comprehensive model.

In this review we have mentioned several renowned multiphonic traditions; discussed the remarkable ancestry of multiphonic sounds; pointed to recent findings; and ensured that examples were given from around the world. But an inclusive and systematic list of traditional multiphonic practices is far beyond the scope of this study. A larger work would first require more explicit acoustic distinctions. For instance, the didjeridu and its "acoustic cousins," the fujara and willow flute should be located in a large category of instruments that produce multiphonics mediated by vocal and tubular resonances. However, within that family these instruments make a unique subset because unlike most members of the larger group—including the orchestral brass and woodwinds—the tubes of these instruments receive little to no modification in the way of tone holes, keys, valves or slides. Finally, we must parse our subset into "cousins" rather than "sisters" because the didjeridu is more fully dominated by vocal resonances — its tube is almost nothing more than a radiator of the performer's vocal sound. This explains the instrument's famously complex and elastic character. By contrast, overtone flutes, like the fujara and willow flute, are more squarely dependent on the properties of the tube. Consequently, they play a more strictly tuned music based on the harmonic series and sometimes require 1, 2 or 3 tone holes to introduce a wider range of pitches. Nonetheless, vocal resonances do direct the melodies of these instruments and give their tones a haunting multiphonic timbre (derived from adjacent members of the harmonic series heard just above and below the focal pitch), which is reminiscent of the didjeridu and other voice-based multiphonic traditions.

A comprehensive study would also address myriad examples, clearly cross-indexed by a variety of factors including material design, performance practices and geographic local. For instance, we mentioned only two overtone flutes of Europe and Russia, and omitted other world traditions entirely. The overtone flutes of Suriname (South America) and New Guinea would surely be an important example in a larger study because they address the limited melodic resource of the instrument by hocketing between two or more performers. This is interesting not only because these appear to be exceptional practices amongst most overtone flute traditions, but also because we couldn't easily use a diffusionist or phylogenetic argument to explain (Montagu 2010) their development.

A fuller survey would have to tackle this thorny issue of transmission more directly. Nattiez's three categories are helpful, but the question of universality, in particular, needs more development. Consistently appealing to recurring material conditions, such as the presence of the voice, a bow, or a tube would be disappointingly coarse. We need to expand beyond universal materials (signifiers) towards universal aims (signifieds). This would give us, for instance, two ways of looking at mimetics, which have certainly produced and reproduced multiphonic traditions around the world. Look no further than the sounds of children (signifier) emulating the bray of donkeys or mules (signified); or the sounds of adolescents (signifier) copying the raspy or gnarled voices of their elders (signified). As we will see, wherever multiphonics thrive, mimetics are rarely far behind. A comprehensive model of multiphonics must be able to seize such universal qualities wherever they appear, and develop them as a starting point, not merely as a descriptive end.

A.2 Multiphonics in Early Classical Music and Jazz

Multiphonics in jazz are well known but they have also been documented in classical music for centuries. Their history there certainly stretches past the written record, but it is probably not by chance that first testimonies come from the 18th century. A little technical background will be helpful.

A.2.1 Benade's first evolutionary period.

Benade (1994) argues that woodwind instruments of classical music underwent three rapid periods of evolutionary change. The first developed in the court of Louis XIV (1638—1715) between the years 1680—1700 resulting in a greater alignment of the response peaks in all the major woodwind instruments. Bringing the natural resonances of the instrument into closer alignment with the first partials of the harmonic series gave these instruments more powerful tone and greater dexterity in speed and register.

These instruments could tackle the new vocal virtuosity of Baroque opera—an influence that had already impacted string writing in the works of Heinrich Ignaz Franz von Biber (1644—1704), Arcangelo Corelli (1653—1713), Giuseppe Torelli (1658—1709) and their successors. The transference was immediate. A rich body of woodwind writing, which included the Brandenburg concertos (1721), was complete by 1730 (Page et al. 2009)

In this context, the first notices of multiphonics begin to appear. At a London recital on January 16, 1729 Joachim Frederic Creta performed the parts of two French horns on a solitary specimen (Blandford 1926). The advertisement points to a gimmick, but that's typical of advertising.[3] The artistic status of three later virtuosos, however, is clear.

A.2.2 First virtuosos.

Giovanni Punto (1746—1803) was universally praised as the greatest horn player of all time. His admirers included Mozart and Beethoven. Punto pioneered hand stopping techniques on the horn and there are numerous written accounts of him performing passages with two, three and four tones at once. In these and other recitals, the purity and balance of his sound was consistently praised (Blandford 1926; Morley-Pegge 2010). Perhaps not coincidentally this virtuoso — who would have been required to improvise, and apparently did so in harmony — was also a violinist and composer.

German bassoonist and composer Franz Anton Pfeiffer (1752—1787) was also known to play three-note harmonies in his solo cadenzas. Pfeiffer pioneered new tone holes to stabilize the upper register and was amongst the keenly respected bassoonists of his day (Waterhouse 2010; Rhodes 1983). Without recourse to his precise instrument and the

[3] Alvin Lucier often recounted David Tudor and John Cage's first performance at Yale. The 1950s concert headline was "Screwball Pianists."(Personal communication, February, 1996.)

quality of reeds he employed, it is difficult to say whether these were sung, collateral or harmonic series-based multiphonics.

Austrian flautist and composer Georg Bayr (1773—1833) was a celebrated Viennese performer. He toured as far as St. Petersburg and developed the "Panaulon" flute together with renowned instrument maker Stephan Koch (1772—1828). The instrument explored a unique low register, which in successive makes eventually reached g below middle C. Bayr's involvement with multiphonics is extraordinary. His book *School for Doublenotes on the Flute* is very likely the first pedagogical text written on multiphonics (Clement 2009; Dick 1975). The multiphonics he used in his recitals shortly after 1810 caused such a sensation that a commission was established in Vienna to determine if a single performer had actually produced all the tones heard by the audience; his virtuosity was scientifically confirmed (De Lorenzo 1951).

Such cases were evidently quite common.[4] Even Berlioz in his *Memoirs* (1933, p. 260) recounts multiphonic feats from the eminent trombonist Herr Schrade and the young French

[4] Dempster (1979) and Trevor (1997) discuss cases of celebrated brass composers and performers from this era straight through to the twentieth century, including Karl Traugott Queissir, Jean-Baptiste Arban, Frederick Neil Innes, Arthur Pryor and Simone Mantia. All of these virtuosos are said to have performed vocal multiphonics as soloists (though they did not employ them in their published compositions or influential pedagogical works.) Details of their techniques are sparse, but their biographies suggest that multiphonics were indeed a wide spread and high profile practice. Queissir (1800—1846) played trombone and viola with the Gewandhaus Orchestra under Felix Mendelssohn. His 1826 transcription of Weber's famous Horn Concertino followed the original work in using vocal multiphonics to create four-note chords during the solo cadenza's adagio ending (Trevor 1997). Arban (1825—1889) was a renowned French cornetist, conductor and pedagogue, who established the cornet's independent program of study at the Paris Conservatoire. Inness (1854—1926) was born and raised in England, where a strong brass tradition of vocal multiphonics persisted

horn virtuoso Eugene Vivier (1821—1900). Yet all of these multiphonic examples seem to fall in the larger category of virtuoso techniques and embellishments that master musicians added to their performances; and there is, generally, a regrettable dearth of documentation on such practices. The Concertino for Horn in E minor, J188, Op. 45 (1815) by Carl Maria von Weber (1786—1826) is unique in this respect because Weber writes out the concerto's virtuosic solo in full detail, unmistakably writing chordal vocal multiphonics directly into the part. The work was premiered by renowned horn soloist Sebastian Rauch and continues to be championed by horn players to this day. Owing to the revival of period instruments, it can readily be heard in several extraordinary recordings (in particular, Goodman 1989).

well into the early part of the twentieth century. Apart from his international tours he worked in the United States from 1874, founding his own band in 1896. Known as "the Paganini of the trombone," Sousa and Clarke considered Innes to be the greatest trombonist of his time. Pryor (1870—1942) was born in St. Joseph, Missouri; already famous as child prodigy he joined Sousa's band in 1892 and was made assistant conductor and director of the band's jazz operations in 1895. He directed his own influential band from 1903 to 1909 after which he turned to conducting, composing and political life. Mantia (1873—1951) was born in Sicily but raised in the United States where he was a celebrated trombone and euphonium soloist with the Sousa and Pryor bands and the Metropolitan Opera (Dempster 1979). Nonetheless, aside from Queissir's 1826 Weber transcription, specific compositions for trumpet and trombone that utilize vocal multiphonics are difficult to identify until the advent of recording, where their presence in jazz is quickly and powerfully documented. An early notated example from the 20th Century is Nathaniel Shilkret's (1889—1982) *Concerto for Trombone* (1945) premiered by Tommy Dorsey and the New York City Symphony Orchestra conducted by Leopold Stokowski (Davidson 2005). The work, and its status as an exemplar, fittingly captures the old and circuitous nature of vocal multiphonics in the brass traditions of classical music and jazz.

A.2.3 Benade's second and third evolutionary periods and the French horn.

Benade's (1994) second evolutionary period (1770—1820) in the history of woodwind instruments focuses on the development of new key systems. Prior to this period woodwinds had six or seven open finger holes, and perhaps one or two additional holes fitted with keys. By necessity these instruments relied heavily on long fingerings (sometimes called cross fingerings or forked fingerings), which often produced heterogeneous tunings and timbral colorations (Figure A.1).

F4:	X X X | X O X
G4:	X X O | X X X
Bb4:	X O X | X X O

Figure A.1: Three common long fingerings on the baroque one-keyed flute. Long fingerings occur any time one or more tone holes are closed beyond the first open hole. (Such notated fingerings always assume a basic movement of left to right, *from* the mouthpiece, or blowhole, *towards* the bell, or foot, of the instrument.)

These instruments were chromatic, but with such mixed results (in terms of intonation, color and dexterity) that many figures could only be realized in particular keys. Between 1770 and 1820 the woodwinds received a number of key mechanisms that eliminated the poorest long fingerings. In many cases musicians added new keys to their instruments in successive modifications, and instrument builders experimented with unique and mutually incompatible designs.

Consequently composers of the period often assumed very different instruments. For instance, all of Mozart's (1756—1791) flute writing can be played on an instrument with six tone holes and one additional hole fitted with a single key, whereas Haydn's (1732—1809) flute writing typically requires an instrument with four to six additional keys. Both composers wrote for a new oboe with smaller tone holes and two additional keys. By the end of this epoch Beethoven (1770—1827) assumed an oboe with ten additional keys – by contrast, modern oboes have at least 45 pieces of keywork! (Carse 1925; Benade 1994.)

In spite of a work like Bayr's *School for Doublenotes on the Flute* most multiphonic fingering systems would evidently have remained the province of individual performers – or perhaps a small circle of performers.[5] Most crucially the sole factor uniting these diverse and piecemeal technical advancements in instrumental design was the pursuit of a new purity in timbre and intonation. This newfound consistency likely mitigated the use of multiphonics on woodwinds.

In 1832 Theobald Boehm (1794—1881) introduced his cylindro-conical flute ushering in Benade's third period of evolutionary change in woodwind instrument design. The details are far too many to recount here, but through the influence of his successive designs all the major woodwinds achieved nearly modern forms by 1847. (This is even true of the saxophone, which debuted in near final form in 1846.) The keywork in these models more than doubled, providing each instrument with primary and neatly sounding fingerings for at least two octaves of chromatic tones.

[5] Two of Bayr's students were said to have mastered his multiphonic system (Clements 2007).

Boehm's 1847 cylindrical flute was the most thoroughly reworked of all the new instrumental designs. He created a tapered head-joint that allowed him to restore the pure cylindrical body of the flute from blowhole to foot. He recalibrated all of the instrument's holes and keys and he engineered massive, consistently sized tone holes that were very close to the width of the bore itself. Thus the first open hole of any fingering provided a ventilation point comparable in size to the tubular ending of the instrument itself – which, when the instrument is fully stopped, provides the nodal midpoint of the instrument's fundamental tone. Moreover, Boehm achieved this while giving a fully vented fingering to nearly every chromatic tone in the instrument's first two octaves (Figure A.2). His 1847 model carried only a single exception, requiring long fingerings for F#4 and F#5 (Baines 1977; Benade 1994). (These universalizing factors — compounded with the nature of the flute's "air-reed" — are the basis of the modern flute's uniquely transposable multiphonics.)

E4:	X X X | X X O
F4:	X X X | X O O
G4:	X X X | O O O
F#4:	X X X | O O X

Figure A.2: Three fully vented fingerings on the Boehm 1847 cylindrical flute and one long fingering. Fully vented fingerings occur when all tone holes are closed up to a specific point, beyond which all further tone holes are open. F# demanded the instrument's solitary long fingering (within in the bottom two octaves.)

Similar work was also completed on crooks and valves for brass instruments. A fully chromatic valve horn systematically developed throughout the 19th century but was not universally adopted until the early 1900s. In an utterly unique development, horn players rejected the development of new keys and instead chose to continue developing the innovations of Giovanni Punto, using their hands to retune pitches available from the harmonic series.[6] While other instruments sought the unique capacity of the vocal cords to produce a pure transition of timbral color across a wide registral range, the horn pursued the timbral and phonetic variety of language itself.

Most of the Classical horn repertoire was written with this color in mind, including the horn concertos of Mozart and Haydn, and Beethoven's Horn Sonata in F Major Op. 17 (1800), which was written for and premiered by Punto. Brahms also wanted this sound for his Horn Trio in E flat Major, Op. 40 (1865). Later still, Saint-Saëns (1835—1921) stated a general preference for the natural horn in all his works (Wick 2001).

Hand stopping persists in the modern valve horn tradition, but not to the same degree. It is notable that within the colorful language of the hand horn style, multiphonics are a relatively modest companion. This no doubt explains why the firmest 18th and 19th century multiphonic traditions rest there.

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[6] Dresden court musician and composer Anton Joseph Hampel (1710—1771) was Punto's teacher and one of the earliest known practitioners of hand stopping. He also developed a set of crooks which kept the horn in steady position relative to the performer so that crook and hand techniques could be combined. The manuscripts of his horn concertos were destroyed during WWII. No known copy survives (Heibert 2010).

The practice also continued into the twentieth century. The hand horn was the principal instrument of instruction at the Paris Conservatoire until 1903 (Wick 2001), and was preserved in the teaching curriculum at other centers until the 1920s. Dame Ethel Smyth's (1858—1944) Concerto for Violin and Horn (1927) added a second fully notated multiphonic concerto to the repertory. Aubrey Brain (1893—1955) premiered the work on a French made hand horn, which had been modified with custom piston valves.[7] In the following years modern valve horn musicians continued to sing the multiphonics of the Weber and Smyth concertos, but they surrounded them with less colorful—though more consistent—melodic lines. Since the 1950s the natural horn has received a significant renewal through the growth of period instrument performances (Meucci and Rocchetti 2009).

Clearly the hand horn tradition couldn't have lasted. As we mentioned earlier, the woodwinds had achieved a well-tuned and evenly colored chromatic scale of at least two octaves by 1847. This was also the epoch during which equal temperament finally became the dominant mode of keyboard tuning across Europe (Barbour 1951).[8] Music literature may

[7] Luedeke (1998) notes that Smyth's Concerto (1927) appears shortly after Kirby (1925) and Blandford (1926) published two articles concerning vocal multiphonics in the *The Musical Times*. While those articles may have provided an impetus for Smyth, it is also true that Aubrey Brain was a noted vocal multiphonics performer, and that his skill was a conventional aspect of the British virtuoso tradition.

[8] Ellis (Helmholtz 1863/1885, p. 549) is an eyewitness to Barbour's study of a century later when he notes that equal temperament first became commercially available in England in 1846 and that "at least eight more years elapsed before equal temperament was generally used for organs."

have demanded these resources earlier in the century, but it now seized them with new and surprising vigor. The harmonies of Liszt and Wagner that lead Lerdahl (2001) to posit a unique "collapsed regional pitch space," certainly required purity of timbre and intonation at the scalar level.[9] The "key" color of the hand horn could not have supported the characteristic parallel relations of hexatonic, octatonic, whole tone, and fully chromatic harmonies that dominated the work of many progressive composers from the 1850s onwards.

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[9] Lerdahl (2001, p. 110) posits a collapsed regional space to analyze the advanced tonal idiom of Wagner and earlier influential mixed mode works of Schubert. In his general model, movement around the circle of fifths is calculated in a "j" variable, whereas changes in the basic pitch class collection are calculated in a "k" variable. If we assume a well-tempered tuning (not to mention a Pythagorean or mean-tone tuning), which becomes progressively more dissonant as it travels outwards around the circle of fifths, then movements at the "j" level also cause an acoustic residue (or distortion) at the "k" level. If the music in question is based on circle of fifths progressions, then this residue simply acts as a kind of extra amplification. Whether or not it is desirable, it is nonetheless consistent. In collapsed regional space, Lerdahl suggests that abstracted scale degrees allow us to posit several "possible" configurations at the scalar level. Thus when observing mixed mode swappings we do not need to calculate new "k" levels. It is clear this system—collapsed regional space—requires something near to equal temperament for such swappings not to introduce a "surplus" circle of fifths-based acoustic residue, or extra "sensory dissonance." Put in simple terms, once equal temperament was firmly established, and most wind instruments had achieved a well-tuned and evenly colored chromatic scale, composers could more effectively disassociate the fifth relations underlying parallel majors and minors, such as C Major and c minor, thereby pursuing new kinds of cyclic harmonies. Put in still simpler terms, without these factors in place the parallel harmonies of Debussy (or Liszt) would not have been strictly—or sounded *strikingly*—parallel. Therefore collapsed regional space has its basis in new ways of thinking about harmony *and* in new sonorous material, obtained progressively between 1770 and 1850 that reinforced the exploration of those harmonic concepts.

A.2.4 French horn postlude.

In a curious article W. F. H. Blandford (1926) draws a firm line between Giovanni Punto (1746—1803) and Eugene Vivier (1821—1900) of three generations later. He derides Vivier's multiphonics as a silly entertainment and compares Vivier to the clown Grock—the famous stage persona of Charles Wettach (1881—1959).[10] Grock performed multiphonics on stage. In one famous number he used ingressive vocal multiphonics to imitate the sound of the double bass, while holding out a pathetic miniature violin (Conant 2009). Luciano Berio (1925—2003) saw Grock perform as a child and molded his *Sequenza V* (1966) around him. The work, of course, employs multiphonics but it is important to note the precise techniques used were inspired by recorded materials given to Berio by Stuart Dempster (b. 1936). And Dempster was unmistakably informed by jazz and new trends in contemporary music (Cope 1989).

As we noted earlier, comedy, language and mimesis have surely evoked multiphonics from time immemorial. All three were alive and kicking in early jazz.

A.2.5 Early jazz.

Imitation of the voice was a quintessential aspect of early jazz. It enabled expressive pathos just as it provided opportunities for high comedy. Early wind players in the jazz tradition "vocalized" their sounds through multiphonics and mutes. Singing below the

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[10] H.S. Edwards (1895) draws a similar portrait, but in more sympathetic language.

instrumental pitch gave them a multiphonic growling sound. Some performers and composers like Joe “King” Oliver (1885—1938) – the mentor of Louis Armstrong (1901—1971) – combined this sound with every mute available. But the king of the growling style was always the plunger. Its material flexibility allowed astonishing approximations of the vocal tract. For many (Dempster 1979; Davidson 2005) the plunger mute is best considered a multiphonic device unto itself.[11]

It is difficult to pinpoint the origin of the plunger mute. Two early musicians who used the plunger are usually credited with the invention: Charles “Buddy” Bolden (1877—1931) and Chris Kelly (1890—1929). There are no known recordings of either trumpeter. However Bolden, suffering from acute dementia, quit playing in 1907. The practice must have been known by then.

The possibility that the hand horn informed the technique is intriguing. It is the only comparable brass tradition, indeed virtuosos in both styles verge on a common sound (for instance Oliver 2006; Goodman 1989). We also know the practices overlapped chronologically — perhaps they overlapped geographically *and explicitly* somewhere in the diverse band genres around New Orleans. The prominence of the hand horn had, of course, spurred the development of additional mutes for the standard brass. So, at least, there is a sure but indirect connection. More pertinently still, former slaves in the region had been

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[11] Being able to single out groups and consecutive members of harmonics, it creates parallel effects to throat singing practices. From an acoustic perspective it turns the bell—the radiating portion of the trombone or trumpet tube—into a subtler resonator with elastic damping properties that can mimic the vocal tract.

brought by the French directly from Senegal and Gambia where the imitation of language through music has powerful traditions (Miller 1995)[12]. Clearly these traditions combined to support the imitation of language on brass instruments. However, short of concrete evidence it may be unwise to push further on either consideration to pinpoint a precise "plunger" origins story. Following Gushee (1995), it is certain that guess work based on notions of an "urban" Bolden or "country" Kelly only limits our imagination of either artist.

Duke Ellington's (1899—1974) orchestra, which took shape over the 1920s, drew on many of the best plunger mute musicians of the day, including trumpeters James "Bubber" Miley (1903—1932) and Charles "Cootie" Williams (1911—1985). Joe "Tricky Sam" Nanton (1904—1946) played trombone in the orchestra until his early death in 1946. He achieved a unique "ya-ya" sound on his horn by combining the plunger and a trumpet straight mute, which could go deep into the bell of his horn. Songs that feature this sound, like *Ko-Ko* and *It Don't Mean a Thing (If it Ain't Got that Swing)*, are still astonishing to hear today (for instance, Ellington 1977a and 1977b). Nanton took the secret of the full range of his

[12] This connection is even more direct than might be supposed. The colonial French felt that agricultural conditions around Senegal and Gambia corresponded closely to those of Louisiana. And so they systematically purchased populations from this region and brought them to Louisiana in large numbers to work the fields. Cultural ties amongst slaves in Louisiana therefore received less than the usual amount of disruption. More typically slaves were gathered at trading hubs in and on the Atlantic with little or no reference to their precise cultural and ancestral background, and they were purchased without such considerations as well.

techniques to the grave.[13] Clearly it involved the coupling of a still little known vocal technique or proportional mixture of upstream resonances.

Growling multiphonic sounds also appeared in the woodwinds. One of the more famous cases was Illinois Jacquet's (1922—2004) tenor saxophone solo on *Flying Home* recorded with the Lionel Hampton Orchestra in 1942 (Hampton 1942). By 1947 Jacquet had gone beyond that performance by replacing the powerful finale of screeching trumpets with his own high pitched squealing sounds (Jacquet 1998). This rough and lightly multiphonic playing style was dubbed the "Texas tenor" sound.

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[13] Jazz trombonist Al Grey (1925—2000) was the foremost plunger mute soloist of his generation; his mute technique was extensively documented with the Count Basie Orchestra and in many small ensembles. Grey developed the "pixie mute" with major mute manufacturers Humes & Berg in order to emulate more closely the restrained vocal timbres of Nanton's playing. Confusion still persists around this device. Grey was disappointed with the steep thickness of the cork used in the mute's final design and emphasized the need to file each cork down to one third or one quarter of its manufactured size (this discrepancy accounting for varying bell sizes in different makes of trombones). He felt this modification produced an appropriately malleable sound, from which one could produce a variety of "vowels" by squeezing the plunger in degrees and developing its position vis-à-vis the bell; however, he insisted that Nanton's famous "ya-ya" contained an additional technique that neither he nor his colleagues had fully identified (personal communication, January 1988). Nanton's plunger mute solo in *It Don't Mean a Thing (If it Ain't Got that Swing)* (Ellington 1977b) supports this position. Without any change in set up, Nanton quickly alternates between two shockingly different "vocal" timbres, with the "ya-ya" sound clearly standing apart from more conventional plunger mute sound. The recording is revelatory because Nanton and Ellington often saved these resources for sectional variations, or used them individually to characterize separate compositions, but here they can be heard side by side. In a final stroke of luck the microphone placement at this 1944 Carnegie Hall recital gave an unusually clear portrait of Nanton's sound (and the entire early Ellington orchestra.) We can hear that the crucial variation is certainly not the small mute in the bell (which Nanton has no time to alter) and that the plunger is largely ornamental to the unique underlying sound it modulates. With this special "ya-ya" technique, Nanton's sound must derive from the vocal tract in a way, or with a proportion, that is still not widely understood and certainly not discussed in any pedagogical or scientific literature.

It first took shape amongst musicians from Missouri and Texas, like Coleman Hawkins (1904—1969) and Herschel Evans (1909—1939), where jazz developed closely with the blues habituating its vocal style and 12-bar form. A second generation of Texas musicians was also identified with the style, including Jacquet, Arnett Cobb (1918—1989) and Buddy Tate (1912—2001). Their playing laid the foundations for the tenor saxophone voice of rock and roll (Miller 1995; Robinson 2010). The "Texas tenor" sound was often the timbral backdrop for more explicit multiphonic and free jazz playing in the 1960s.

A.3 Multiphonics After 1950

The diffusionist explosion of musicians and trends incorporating multiphonics from the 1950s onwards makes sketching a short history of the period impractical. Writing in 1977 Gerald Farmer identified 79 composers working with multiphonics (Farmer 1977). In retrospect his list of classical composers was already sorely incomplete and beyond this he failed to mention a single jazz musician — had he done so, the list would have been much larger still. Today the problem is compounded. The possibilities of such a catalog grow with each passing year. What we can do with the space provided is improve the picture of earliest efforts in this period, and use those cases to articulate a few key issues.

A.3.1 Landmarks and reservations: 1957—1959.

It is convention (for example, Farmer 1977; Salzman 1988; Watkins 1988) to state that woodwind multiphonics were first written by Luciano Berio (1925—2003) and Franco

Evangelisti (1926—1980) in their respective flute solos *Sequenza I* (1958/1992) and *Proporzioni* (1958). They *might* be the first woodwind compositions to provide written fingerings for their multiphonics, but any further attribute is problematic. Certainly the earlier portions of this chapter summarily dismiss the common use of these solos as a "multiphonics origins story."

Nonetheless, something new seems to have been in the air. John Cage (1912—1992) called for multiphonics in two scores written one year earlier in 1957. He asks for "undertones" in his 1957/1958 *Solo for Clarinet* (Rehfeldt 1977) and "intervals" in his 1957/1958 *Solo for Flute* (Pritchett 1996). And John Coltrane (1926—1967) began writing multiphonics still earlier that same year. Initially he used them in lead sheets (the written portion of a score that opens and closes the performance), where he felt certain to perform them correctly, because they'd be played the same way each time (Porter 1998). Coltrane recorded his first multiphonic lead sheet on May 31, 1957.

In *While My Lady Sleeps* he employs two multiphonics supporting parallel thirds to close the final repetition of the principal melody. A soft cymbal roll is added to mitigate the roughness of the sound. The recording was released in 1957 on the Prestige album *Coltrane* (Coltrane 1957). Already famous from his work with Miles Davis (during 1955-1957) and Thelonious Monk (in 1957), this was Coltrane's bandleader album debut. Later he employs a similar ending for the ballad *I'll Wait and Pray* recorded in 1959. Two other works from that session – *Harmonique* and *Fifth House* – also use multiphonics but in very different ways; all three compositions were released together on the 1961 album *Coltrane Jazz* (Coltrane 1961).

Harmonique is a triple time blues with three multiphonics in the principal melody. Coltrane explores two of these during the opening of his solo. *Fifth House* is a unique case, which has escaped detailed commentary. The form is made of two eight bar sections, organized AABAA. Coltrane continuously outlines *Giant Steps* changes, but in the A sections the rhythm section accompanies him with a simple modal harmony sketched by a funky three-note syncopated figure emphasizing every fourth beat. At the B sections the rhythm section jumps into the *Giant Steps* changes with him, suddenly emphasizing a fast swinging time right at the level of the pulse.[14] Coltrane plays the A sections with a leaping line and widened embouchure, letting notes crack and split into subtly multiphonic sounds. Although he's still playing *Giant Steps* changes, the modal background (and no doubt the slow and funky vamp) seems to give him license to merge timbre and harmony more freely exploring rough and sometimes ambiguous sounds. When the B sections arrive his embouchure tightens and he carefully traces his speedy line keeping all notes speaking perfectly. Presumably, because the multiphonic intrigue happens in short subtle moments that seem like rough attacks, shaky sustains or grace notes, the track has been completely missed in the development of his multiphonic work. But this is definitely the root — in its youngest

[14] Coltrane used his *Giant Steps* chord progressions in a number of compositions. All of these works explore a rapid series of hexatonic changes elaborated with previous dominants. For instance, in Coltrane's composition *Countdown* the standard four bar progression | <u>ii7</u> | <u>V7</u> | <u>IMaj7</u> | <u>IMaj7</u> | becomes | <u>ii7 – V7/bVI</u> | <u>bVI – V7/III</u> | <u>IIIMaj7 – V7</u> | <u>IMaj7</u> |.

recorded form — of Coltrane's famous 1961 solo on *Chasin' the Trane* (Coltrane 1997) and the more explicit multiphonic playing that followed (for instance Coltrane 1963, 1965, 1966a, 1966b, and 1967).[15]

The works of *Coltrane Jazz* point forward to a basic divide amongst many multiphonic practices: the tendency to seize multiphonics primarily as timbres, or to seize them primarily as harmonies. The appropriateness of either emphasis (much less the means of proceeding down either path) can still be a contentious issue. *Fifth House* points to the timbral tradition that blossomed more prominently in Coltrane's circle and in the freer harmonic styles of musician's working in East Coast and Midwestern states (Mingus 1960; Dolphy 1964 and 1999; Ayler 1965; Kirk 1965/1967; Braxton 1968), while *Harmonique* points to the "harmonic" approach developed more fastidiously by Bert Wilson (b. 1939) and other north West Coast performers (Zitro 1967; Bergeron 1989; Wilson 1997 and 2005; Gross 1998; Phelps 1998).[16]

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[15] Coltrane's famous *Chasin' the Trane* solo is a marvelous conflation of series-based multiphonics, collateral multiphonics and the 'Texas tenor' sound — itself a form of vocal multiphonics. Like the modal harmonies of *Fifth House*, the 12-bar blues often proved to be a powerful non-binding harmonic background for free jazz playing. Ornette Coleman (1958 and 1961) explored this intersection in a post-Parker be-bop style largely devoid of multiphonics. Later Albert Ayler (1965) picked up the subject again, bringing aspects of both these approaches together (Jost 1975; Heffley 1996).

[16] Gross recounts listening to *Harmonique* with Bert Wilson in 1961. Astonished with Coltrane's technique they quickly began searching for other multiphonics they could "fit" into the chord progressions they were playing (Gross 1998).

A.3.1.1 Interlude: narrative problems.

An appropriately poised story of these multiphonic exploits is hard to find. Zita Carno, longtime keyboardist of the Los Angeles Philharmonic, wrote the original liner notes for *Coltrane Jazz* (1961). She details the multiphonics in Coltrane's lead sheet and solo for *Harmonique*, and his use of high multiphonics at the close of *I'll Wait and Pray* (Carno 1961). (She does not, however, mention *Fifth House* or the 1957 multiphonics of *While My Lady Sleeps*.) Yet it has come to be assumed that Coltrane wrote lead sheet multiphonics only once, for the 1959 *Harmonique*, and that he didn't begin improvising with multiphonics until 1960 and 1961. Even the 2001 reissue notes of *Coltrane Jazz* (1961) repeats this mistake: "Coltrane's excellent if typical solo [to *Harmonique*] pays no attention whatsoever to this timbral technique. [His use of multiphonics in the melody] could have faded into memory as a parlour trick, if it hadn't forecast what would soon become one of Coltrane's sonic trademarks" (Tesser 2001).

Ironically, many jazz writers dismiss the multiphonics of 1957 and 1959 (where they notice them) because they were simply written (although, as we have noted, Coltrane did improvise with multiphonics in this period too). While at the same time most classical writers dismiss the work (that is, what they know of it) because they assume it's simply improvised. For instance, writing about the 1958 solos of Berio and Evangelisti, Farmer (1977) notes, "[they] are perhaps the first examples of notated music that attempted to extend the technical possibilities of woodwind instruments to include [multiphonics and other new devices]…in the jazz field bass clarinetist Eric Dolphy and saxophonist John

Coltrane were among the earliest to employ multiphonics. Although there are no notated examples of their use of their technique, they continued to experiment with a vast array of sound combinations until their deaths in the mid-1960s." This précis is unacceptable, in part for missing the chronology, but mostly for refraining from commentary and reflection because the work has no "notated examples". Notation is important because it provides us a document to study. However, recordings are important documents too. A Medievalist would drown in his own drool for the documentary wealth surrounding multiphonics in the 1950s and 1960s. In a more important text Glenn Watkins (1988) is no better, he simply mentions the case of Berio. Salzman (1988) does the same. Perhaps more worryingly, these stories also find their way into jazz scholarship.

In his excellent biography on John Coltrane, Lewis Porter is the first to detail the appropriate timeline for Coltrane's involvement with multiphonics, beginning in 1957, but he misses the larger picture. Porter opens his discussion of multiphonics like this: "multiphonics (literally, many sounds) is the technique of producing more than one note at a time on a woodwind instrument…modern composers of chamber music have studied their effects and jazz people have picked them up." (1998, page 120). A better twentieth century music history is desperately needed.[17]

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[17] Austin (1966) may still be the best model for a supple movement between classical music and jazz where demanded by topic. Although his style is often outdated (witness his essay on Poulenc), he doesn't make the common mistake of saving jazz topics for later, and he doesn't hesitate to introduce transcriptions where needed. Salzman (1988), by contrast, is typical. His compartmented topics lead him to discuss third stream composers before introducing jazz itself. When he does explore jazz he focuses on "first" masters, without touching on the figures that would finally explain the earlier "third stream" chapter.

A.3.2 1957—1959: Musicians.

Although we have improved the timeline for 1957-1959, our general history points to the much larger issue that multiphonics have long been known by masterful and amateur musicians alike.[18] Coltrane, Cage, Berio and Evangelisti all relied on musicians who knew multiphonics well. Italian flautist Severino Gazzelloni (1919—1992) best explains the appearance of multiphonics in the two 1958 Italian flute solos. Both compositions were expressly written for his 1958 Darmstadt recital, and Gazzelloni had worked directly with each composer providing them with intriguing instrumental techniques; Gazzelloni later recorded these and other notable premieres on his celebrated 1962 album *Music for Flute | Music for Flute and Piano* (Gazzelloni 1962).

Cage collaborated with a specific musician for each of the "solos" in his *Concert for Piano and Orchestra* (1957/1958). This explains the mixed terminology in different instrumental parts. The performers were exceptional (Pritchett 1996). For instance, Andrew Lolya was the principal flutist of the New York City Ballet for thirty years and Frank Rehak

[18] On this Rehfeldt (1977) wryly assures us about the universalist transmission of multiphonics, pointing out that as long as young professionals have diligently pursued the upper registers of their horns, multiphonics have certainly been known.

was a leading be-bop trombonist of his generation; all of the musicians were experienced in classical music and jazz.[19]

John Coltrane heard Philadelphian saxophonist John Glenn playing three and four note chords and asked him how it was done. Glenn possessed a versatile command of multiphonics, and explained the basics to Coltrane and one of his students. The story goes that the student caught on first, but Coltrane could soon play them on all the notes of his horn (Porter 1998).[20] These stories fit our history well. Musicians have long known multiphonics.

[19] The "solos" point to an open and fruitful collaboration. It is true that Cage said they played like idiots (Pritchett 1996), but the recording is—for the most part—still compelling (Cage 1994). Likely he would have been more kind in his remarks if tuba player Don Butterfield hadn't quoted the unmistakable "Db-Bb-Eb-Bb" ostinato from Igor Stravinsky's *The Rite of Spring* which famously presages the pounding "Dances of the Young Girls." Some visual antics also seem to have contributed a degree of unintended humor to the performance, but it is difficult to tell who was doing what.

[20] The development of Coltrane's multiphonic playing is also indebted to Eric Dolphy (1928—1964). Dolphy already had an elastic command of multiphonics in *I Wish I Were in Love Again* (Dolphy 1959). Unfortunately, the earlier recordings are too sparse to deduce the history of his multiphonic development. After a few early 1948 recordings his playing goes undocumented until 1958 when he appears suddenly as a fully formed and uncommonly versatile musician (Kernfeld 2010). His solo in *What Love* (Mingus 1960) explores collateral, harmonic series-based and vocal multiphonics in a theatric "musical dialogue" with composer, bassist and bandleader Charles Mingus (1922—1979). Perhaps no other recording more fully anticipates the multiphonic styles that would develop during the 1960s; and this comment certainly extends to the theatrical multiphonic-based instrumental works of contemporary music, such as Salvatore Martirano's (1927—1995) *Underworld* (1964/1965) and Luigi Nono's (1924—1990) *A floresta é jovem e cheja de vida* (1965/1966) (Bish 2005). When Berio (1985) called *Sequenza V* (1966) his "most imitated composition" he managed to overlook this entire tradition and the many relations composers already had, and would have, to it. But perhaps Dolphy was oblique about Berio as well. His original composition *Gazzelloni* (Dolphy 1964) is named after the Italian flautist (whom Dolphy might have met

A.3.3 1957—1959: Further influences.

It is tempting, then, to ask what factors led to this bundle of activity and it's unprecedented diffusion and development in subsequent years. The development of electronic music was surely a crucial factor. Between 1951 and 1955 studios for electronic music launched in Paris, Cologne, New York, Milan, Tokyo and Eindhoven. The mimesis of – or dialogue with – electronics gave multiphonics an entirely new context. Both Berio and Evangelisti created tape compositions in the years immediately preceding their flute solos. The influence is obvious in Evangelisti's score; Berio's work is more restrained in this regard, using only two multiphonics (each once) in a consecutive and strictly harmonic context. Cage had a longer history with electronic music that stretched back to *Imaginary Landscape No. 1* (1939). But still more instructive for his solos are the muting effects, inharmonic resonances, and noise-based sounds of *Bacchanale* (1938), Cage's first work for prepared piano. His indeterminate requests for "undertones" and "intervals" in the 1957/1958 solos carry no remarkable features likening them to speech or electronic sound. Rather, they stem clearly

earlier that year), but Dolphy's solo is a studied dialogue with Berio's *Sequenza I*. Both works explore contrasts of speed and stasis, and the way rapid articulations can create an ambiguous space between the two. In *Sequenza I* Berio's two fleeting multiphonics create yet another manifestation of this ambiguous transition between stasis and dynamic polyphonic materials. In *Gazzelloni* Dolphy uses two small multiphonics in precisely the same way. Commentators focus on the simple presence of Dolphy's flute multiphonics, but these brief intervals—and the fact of them being on flute—are nothing in the general course of Dolphy's multiphonic playing. Their poise, however, is unique. Moving beyond the parallel to Berio's score, Dolphy also ties his two small multiphonics to a "metric crisis" in the rhythm section that exploits a basic ambiguity in the work's form, vacillating as it does between "A [(4+4)+(4+4)] B [4+6]" and "A [4+4+4+4+4] B [4+2]".

from the extended techniques tradition of his prepared piano and his wider aesthetic embrace of sound for it's own sake.

The inharmonic resonances of the prepared piano liken it, or perhaps even classify it, as a multiphonic device. But more crucially, the instrument as a whole reflects the century's sustained dialogue with noise. The mimetics of noise (and a residual tolerance for noisy sounds) acquired new dimensions at the beginning of the century and established an enduring backdrop for the development multiphonic sounds. The subject is a book of its own, but a few examples here will point to the range and presence of the phenomenon. Luigi Russolo's *The Art of Noises* (1913) simply laid the conceptual project bare (Antokoletz 1992; Morgan 1998; Tisdall 1977), while Igor Stravinsky's *The Rite of Spring* (1913) demonstrated the staggering power and potential appeal of raucous and noisy sounds (Eksteins 1989; Morgan 1994). The following year Henry Cowell performed "inside the piano" at his second San Francisco recital in 1914. In one stroke he produced a new inharmonic or multiphonic tradition and set the stage for a century of noise-based interventions that ran roughshod over many of the instrumental refinements we have charted from the 1680s onwards. Of course, at the time Henry Cowell was just an outsider, and a teenager to boot (Rich 1995). But that same year, Leopold Stokowski was named music director of the Philadelphia Orchestra: now noise had an establishment champion ready to promote the work of Stravinsky and Varèse, and later Cowell himself, along with the first electronic music compositions of Otto Luening (1900—1996) and Vladimir Ussachevsky (1911—1990), billing them all as vital public spectacles (Smith 1990).[21]

In short, by the late 1950s multiphonics had renewed and powerful foundations in the mimetics of language (through early jazz); in the mimetics of noise (which also spurred the broader exploration of extended techniques); in the mimetics and technical discoveries of electronic sound; and in the work of a growing rank of masterful performers and composers who ultimately made multiphonics more widely known as a phenomenon unto themselves. In the following two decades, world music traditions, lexical multiphonic catalogues and theoretical studies would broaden the contemporary dialogue with multiphonics still further.[22]

[21] Although Stokowski may have had more subtle successors, it is notable that noise, electronics and multiphonic sounds continued to have public champions in free jazz and rock and roll. As we have seen, the work of John Coltrane first piqued the theoretical inquiries of physicist Arthur Benade.

[22] World music traditions were arguably a powerful influence on multiphonics in the 1950s and before. We have discussed their roots in early jazz. They also informed the instrumental techniques and philosophical dispositions of Cowell, Cage, Lou Harrison (1917—2003) and others (Rich 1995). In the field of jazz the same is true of Yusef Lateef (b. 1920) who began exploring vocal multiphonics and a wider variety of world music influenced extended techniques on the flute as early as 1957 (Lateef 1957).

Appendix B: Key Terms and Issues for the Study of Multiphonics

B.1 Combination Tones

Combination tones have a vexed history that confuses their meaning and applications. They are important for understanding multiphonics and yet, at the same time, their significance can easily be overstated. Let's sort them out.

Theoretical writings don't hone in on multiphonics until early in the 20th century (though we'd love to see the Viennese commission on Bayr's 1810 multiphonic recital clearing him of fraudulence!) Combination tones, however, were studied much earlier. Their theoretical history begins in the 18th century when they were independently observed and reckoned by the German composer, organist and theorist Georg Andreas Sorge (1703—1778) sometime between 1745 and 1747; the French scientist Jean-Baptiste Romieu (1723—1766) in 1751; and the famed Italian composer, violinist and theorist Giuseppe Tartini (1692—1770) in 1754. (Buelow 2010; Lindley 2010; Petrobelli 2010). The first order difference tone ($f2 - f1$), which Tartini recommended as a method for tuning double stops, eventually took on his name becoming known as "Tartini's tone" (Johnson 1985).

Combination tones are additional notes that appear when two pitches are combined. Under ideal conditions the math needed to calculate these tones is simple. We add and subtract the frequencies of the two or more driving tones to find the frequencies of the combination tones. For instance, two simple sinusoidal waves $f1$ and $f2$ (where $f1$ = 200 Hz and $f2$ = 250 Hz) will combine to create the difference tone $f2$ - $f1$ = 50 Hz, and the summation tone $f2 + f1 = 450$ Hz. These sums and differences can be produced at multiples of the original driving signals as well, thereby creating further tones at $2f2 - f1$, $2f2 + f1$, $f2 - 2f1$, $f2 + 2f1$, $2f2 - 2f1$, $2f2 + 2f1$, $3f2 - 2f1$, $3f2 + 2f1$ etc.

It is important to note that combination tones occur in nonlinear resonating systems, where they are considered distortion products, but that some combination tones can also be perceived in the open air through linear constructive and destructive interference when two or more instruments play at the same time. Much speculation has been raised about how we hear difference tones produced in the open air because there is no unique power observable in the air at those frequencies. For instance, take our two simple sinusoidal waves $f1$ and $f2$ (where $f1 = 200$ Hz and $f2 = 250$ Hz). Sounded separately they would combine in the air creating the impression of a difference tone at 50 Hz. But a frequency analysis of the situation would show only two energy levels present, one at 250 Hz and one 200 Hz. Contemporary research now shows that pitch perception, for all but simple sinusoidal tones, is heavily based on repetition rate or in some cases envelope repetition rate recognition (Jülicher, Andor & Duke 2001). This explains the 50 Hz tone, which is clearly the largest envelope-shaping factor in the signal but has no unique energy of its own: it is merely the periodic powerful sum of amplitudes from the other signals. From the perspective of envelope repetition rate, the ear doesn’t need to create a distortion product to recognize this amplitude artifact, though if the signal was loud enough, or at the right frequency, the basilar membrane could certainly create such a tone through its own processes of nonlinear distortion. Let’s look at nonlinearity more closely.

B.2 Nonlinearity

Any material or junction that produces an output physically different from the sum of its inputs is considered to be nonlinear. From a material perspective, such distortions are said to arise when energy passes through a medium with an uneven distribution of pressure gradients. Nonlinear systems can be quite different from one another. Some have stronger or weaker effects, consistent or chaotic effects, but in all cases they are neither linear nor random systems. Most nonlinear systems have more than one nonlinear component. In practice, almost all materials are at least slightly nonlinear, so mathematicians, researchers, and musicians mostly concern themselves with instances where those results are pronounced.

It is helpful, briefly, to consider how mathematics corresponds to physical structures underlying nonlinearities. Figure B.1 presents the basic equation for a simple harmonic oscillator where m, R and K are constants:

$$m\left(d^2y/dt^2\right) + R\left(dy/dt\right) + Ky = F(t)$$

Figure B.1: The basic equation for a simple harmonic oscillator. The variables in this equation are: m=mass, R=damping coefficient, K=spring coefficient, d=distance, t=time, y=displacement, and F(t)=applied force. For harmonic oscillation the coefficients m, R and K must be constant.

If coefficients m, R and K are constant, then the system described in the equation is linear. However, the response of a spring (the spring coefficient) does not remain constant when it is pushed to extremes; and damping behavior does not remain constant in the presence of turbulence and other sometimes more complex factors. Consequently, nonlinearity arises as one or more of the coefficients m, R or K comes to depend on the dependent variable y (displacement).

In musical acoustics we can largely assume that m is constant (in other, exotic settings it could be variable.) However, if we take the example of a clarinet or saxophone, we see that spring activity of the reed and damping activity of the body are rarely constant. The reed has uneven density and is fixed at one end; its response is inherently variable. The body of the instrument may have an approximately ideal shape, but joints and keyholes (opened or closed) introduce turbulence. Therefore most musical instruments contain multiple nonlinearities, which contribute to standard and nonstandard tone production alike.

It is commonly assumed that nonlinear distortion is inharmonic. However, if two frequencies $f1$ and $f2$ are harmonic and drive further nonlinear products, then all their distortion products will also be harmonic. For instance, given $f1$ = 200 Hz and $f2$ = 300 Hz, then $f2 - f1$ = 100 Hz and all higher order combination tones will produce multiples of this. Further it should be noted we have generally been discussing two-term nonlinear distortion (which can be harmonic or inharmonic). There are other kinds of distortion including one term harmonic nonlinear distortion, which given $f1$, can produce $2f1$, $3f1$, $4f1$, $5f1$ etc. However, as has been stressed throughout, these are deep processes. It is the way such processes combine that creates particular and familiar timbres.

Let us consider two common examples of nonlinear distortion in the domain of analog electronics, taking one in greater detail. We'll then reflect on two cases of nonlinear distortion with biological and physical origins.

B.3 Amplitude Modulation and Frequency Modulation

The analog ring modulator uses a loop of diodes (a common and conventional nonlinear electrical component) to modulate the amplitude of one input signal called the carrier (which is heard directly in the output) according to the frequency and amplitude of another input signal called the program or modulating signal (which is not placed independently in the output.) If the carrier is a 150 Hz sine tone and the program is a 90 Hz sine tone, then the output will contain the carrier signal (150 Hz) and two further tones: the sum of the carrier signal and program signal (150 Hz + 90 Hz = 240 Hz) and the difference of the carrier signal and program signal (150 Hz – 90 Hz = 60 Hz). Each of these tones, the carrier and the sum tone and difference tone (which are often called sidebands) will have unique energy of their own. That is, if we perform a frequency analysis of the ring modulator output in open air, we would observe unique energy sources at the frequencies of the carrier, the sum and the difference.

The ring modulation carrier can be more than a simple sine wave. For instance, it can be a complex tone with harmonics (such as a quiet or high tone played on the flute, which may happen to have three partials). In this case a pair of sidebands (a sum and a difference tone) is created for each of those partials (Figure B.2). This is its essential mode of operation.

Third Set of Sidebands:	($3f1 – f2$ = 2550 Hz) and ($3f1 + f2$ = 2730 Hz)
Second Set of Sidebands:	($2f1 – f2$ = 1670 Hz) and ($2f1 + f2$ = 1850 Hz)
First Set of Sidebands:	($f1 – f2$ = 790 Hz) and ($f1 + f2$ = 970 Hz)
The Carrier:	($f1$ = 880 Hz, $2f1$ = 1760 Hz, $3f1$ = 2640 Hz)

Figure B.2: Ring modulation output sample. Where the carrier signal is a simple flute tone with three harmonics ($f1$ = 880 Hz, $2f1$ = 1760 Hz, $3f1$ = 2640 Hz) and the program or modulating signal is a 90 Hz sine tone ($f2$ = 90 Hz). Note that the program signal ($f2$ = 90 Hz) is not present as an isolated frequency with unique energy (amplitude) in the output signal.

Frequency modulation (FM) also uses a pair of input signals to produce a still richer sonority built of sum and difference tones—which also have objective presences (unique energies) in the output signal. However, in FM modulation the *frequency* of the carrier signal is acted upon by the frequency and amplitude of the program signal. The number of sidebands created and the relationship of their amplitudes (both a factor of the modulation index) is significantly more complex than in amplitude modulation, but these elements can be readily calculated all the same.

Such analog electrical devices (and their digital counterparts) are relatively stable. Their primary effects can be smoothly produced up and down the frequency range. Depending on what controls are provided with the instruments or interfaces, it is possible to manipulate their components individually after the act of processing them. For instance, the output volume of the carrier signal on a ring modulator can be turned down completely so that only the sidebands (the sum and difference tones) are heard; one can also filter out the sum or the difference tone so that finally only a single frequency (the sum of difference) is heard. These are essentially post-process mixing operations, yet they do make clear the objective nature of the output sounds. Because these sounds have unique energies at their frequencies, surrounding sounds (post-process) can be filtered away leaving any one tone to be heard independently.

These modulation processes have further sounds lurking inside them—sounds we can perceive even though they have no unique energy. One thing that makes them very different is that there is no way to isolate them post-process. If we filter away the other tones, these sounds will go with them.

What are these sounds? We will discuss them below in detail, but if we look at our ring modulation example again (Figure B.2), we can get a picture. Notice both terms in each pair of sidebands will produce a 90 Hz difference tone in relation to some part of the carrier signal.[1] Also, if we take any pair of sidebands and subtract the lower term from the higher term we produce the difference tone 180 Hz, which forms an octave with the 90 Hz difference tone. Thus Figure B.2 shows three pairs of sidebands reinforcing a 180 Hz difference tone, and six pairs of waves (each containing one sideband element and one carrier element) reinforcing a 90 Hz difference tone. The problem is no part of the process is giving these difference tones unique energy. They are observable amplitude repetition rates in the signal (occurring at 90 Hz and 180 Hz), which arise from the composite energies of

[1] If the math isn't immediately clear, it will be. Consider the first set of sidebands ($f1 - f2$ = 790 Hz and $f1 + f2$ = 970 Hz) in relation to the first component of the carrier signal ($f1$ = 880 Hz), it follows that: $f1 - (f1 - f2)$ = 90 Hz and $(f1 + f2) - f1$ = 90 Hz.)

other signals. Our ears, however, observe salient repetition rates, and because these are very clear, we hear these tones. Can these sounds take on real frequencies of their own? That is a complex question, but if you put the output of the ring modulator into two ring modulators with program signals at 90 Hz and 180 Hz, they will produce these tones *and more* with very real energies.

B.4 The Cochlea

The ear is also a nonlinear device that produces objective combination tones, but its behavior—in contrast to the previous examples of AM and FM modulation—is very difficult to model. For instance, when the ear receives two input signals it may or may not produce noticeable distortion products depending on the volumes and frequencies of the input signals. It is also a *complex* nonlinear system with a variety of nonlinear components and functions. These functions go beyond the active creation and amplification of energies, to include the passive pre-filtering of energies as well (Cho 2000; Jülicher, Andor & Duke 2001).

For instance, when a signal that is rich, with frequency clusters in various positions, approaches the cochlea, the basilar membrane passively pre-filters the frequency clusters. Specifically, receiving areas on the basilar membrane (for specific frequency regions) respond to tightly knit collections of frequency energies with a generalized hair bundle motion. (This first layer of distortion, which is a passive prefiltering, is a root part of the process underlying the phenomenon known as critical bands and it is an important factor in clarifying our perception of repetition rates.)

The pre-filtered representation of the sound now vibrating in the cochlea undergoes the selective active amplification (originating from individual hair cells) and the generation two-term distortion products—usually described as difference tones at successive orders, with strong preferences for cubic terms (i.e. $2f1 - f2$) and quadratic terms (i.e. $f2 - f1$). As in the case of AM and FM modulation, these distortion products are indeed objective energies. (These frequencies, created and amplified directly by the ear, are called otoacoustic emissions and can be precisely measured via frequency analysis.) However in a profound contrast to AM and FM, these objective energies—once established—are as real as any other received tone on the basilar membrane, and therefore produce still further terms. It seems the ear can input its own output.

Also in contrast to AM and FM modulation, the active processes described above—active amplification and the generation of two-term distortion products—regularly interfere with each other in a nonlinear fashion. Put simply, the introduction of two-term distortion products (and noise itself) in particular regions suppresses the active amplification process in precisely those regions. This interference is a root cause in the phenomenon of masking (Jülicher, Andor & Duke 2001).

Before concluding let's consider some specific implications for multiphonic research. It is important to note that frequency analyses of multiphonics, especially rich ones, are often difficult to interpret. They present terms that are cancelled out by passive pre-filtering and slight intensities in the analysis may receive active amplification while others may be ignored through the interference of masking. Finally the "cleaned up" model in the cochlea receives further attention. Additional two-term distortion components will likely be added that are not present in the frequency analysis. Envelope repetition rate detection will look for salience—and some of those saliences will be more remarkable in the ear's final version of the sound. As stated before, some salient envelope repetition rates might have no energy of their own and therefore go totally unreported in the frequency analysis. This last factor likely explains our perception of a number of multiphonics, most notably on the oboe, that produce sensations of fundamentals below the frequency range of their instruments. (Where the instruments themselves can produce no unique energies.) Because these sounds 1) can be heard in recordings played back at low volumes, 2) resist masking when doubled by other instruments, and 3) occur where envelope repetition rate is most salient (at the greatest common denominator), it seems awkward to describe them as active process two-term distortion products created by the ear. (However in some cases, for instance if the signal is very powerful, the ear certainly produces those products as well.)

Short of scientific evidence to cite, extensive practical experience suggests it is the *global or generalizing* interpretations of complex multiphonics, resulting from pre-filtering, masking and envelope detection, that are hardest to interpret when looking at frequency analysis. The problem of not finding individual distortion products a*dded* by the ear is more rare. In a similarly way it is also difficult, when looking at frequency analysis and traditional harmonic transcriptions of multiphonics, to interpret or anticipate the global aspect of timbre, which in the case of multiphonics can go beyond color, becoming something closer to “stress” or “duress” in the sense of Bregman’s scene analysis (i.e. does this sound reveal an object that is about to break?)

Abstracting from all of this, perhaps the greatest difference between nonlinear functionality in the ear and in analog and digital AM and FM, isn’t the presence of a one attribute like pre-filtering, it is that the ear is biological. Nonlinear distortion arises according to the ear’s resonant properties, the thickness and thinness of the cellular mass, the length and responsiveness of tiny hairs, the range of motion made available by the physical construction of system. The ear is beholden to unique physical nonlinearities—starting with complex dependencies in spring and damping coefficients described in Section B.2—it hasn’t been electrically engineered to do what it does in a smooth continuous way. (Quite the contrary, we assume it evolved certain specific contrasts, like amplitude responses in particular frequency ranges for survival.) It also does not calculate its actions (like a digital ring modulator) so much as it simply does them (Noë 2004).[2]

Therefore, it is easy to underestimate the mathematics needed to predict and describe a biological resonating system like the ear. For powerful accounts of these issues see Jülicher, Andor & Duke (2001) and Knuth (1998). In his article Knuth stresses that

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[2] Researchers used to argue as to whether or not a mental form of frequency analysis was responsible for the generation of virtual fundamentals, now researchers assume they are perceived directly as a first order phenomenon without any need to “double check“ for discrete power. In *Action in Perception* Noë (2004) charts similar reversals of theorizing in allied forms of perception to put forth a new philosophical and cognitive model of the mind, which emphasizes the primacy of “doing” wherein action and observation are sharpened by embodiment and top-down considerations rather than a flat and high level of neural processing.

calculations which utilize the basic math of the sums and differences of frequencies without taking into account, at least, the amplitudes of the input signals and basic multiple nonlinear resonances of the ear produce descriptions impoverished in essential respects. He shows that progressively taking into account more detailed factors can easily lead to the complete cancellation of low order combination tones such as 2f2 – f1 while simultaneously giving rise to pronounced higher order combination tones such as 3f2 – 2f1.

More importantly he demonstrates that careful analysis of distortion products can reveal the physical nature of the nonlinearities in the system itself, what varieties there are, how much variability they have, and if there are multiple nonlinearities which share or divide basic functions observed. Ultimately such a comprehensive description (a richer and more idiomatic algorithm than given in Figure B.1) could contribute to building a physical model of the ear and its distortion products, or an instrument and its multiphonics. (And such a model could be extended hypothetically to imagine the effects of innovative alterations to the device.) In short, a characteristic description must go beyond relating the type of process (frequency modulation patterns), to relating specific manifestations or trends in the process. With that step we could have a distinguishing description that allows us to model the system itself.

B.5 The Nonlinearity of Musical Instruments

Though musical instruments are primarily intended to produce sounds and the cochlea is primarily intended to receive sounds, they do reveal some broad similarities. They are both complex physical bodies and often times the multi-resonator systems of musical instruments include resonators, which like the ear, are biological (cane, the voice, the lips, the vocal tract). In short, musical instruments are also complex constructions with various nonlinearities. Consequently their resonances and the two-term distortion products they support shift qualitatively and quantitatively up and down the instrument. Like the basilar membrane, the bodies of instruments (the lightly damped resonators) pre-filter sounds, which, in the case of musical instruments, often originate from heavily damped resonators in the simple system. Also like the ear, nonlinear distortion in the system can become stable and go on to inform further combination tones with objectively measurable energy. For instance, take two driving tones A and B, in an instrument such that the distortion product (A + B) falls in a powerful formant frequency. In this case, it is no longer adequate to describe further distortion products such as (2A + 2B) as having arisen from simple descriptions of components A and B, because the amplitude of (2A + 2B) will be due, in significant degree, to the unique power of (A + B) at the formant frequency.

The nonlinearity of musical instruments is rich and like the ear needs a description that goes beyond the presence of combination tones to arrive at a description of the underlying physiognomy itself. Unfortunately there is no pedagogical work for musicians that approaches instruments in this way. Orchestration books could be a reasonable place to start the discussion. However, Piston (1955) aside—which is notable for introducing musical instruments together with basic acoustic data such as the metrical lengths of their fundamental frequencies—most standard orchestration books (Adler 1982; Blatter 1980) follow Rimsky-Korsakov (1922/1964) and Forsyth (1935) in presenting the instruments as a rich cast of characters rather than acoustic entities. This is true even of Blatter (1980), which is otherwise notable for embracing contemporary technique with zeal. Surely this is one reason that multiphonics and other extended techniques often seem unassimilated when appearing on the surfaces of compositions. While the thrust of these texts is sound, the approach is ready for augmentation. For English readers the 2009 translation of Meyer (2004) was a breakthrough. The work discusses column lengths, bore properties and formant frequencies in detail when introducing each instrument. However, it does not address multiphonics and extended techniques directly and it is not conceived as an advanced instrumentation and orchestration text for musicians and composers; it is, more essentially, a highly theoretical resource for audio engineers. On the side of acoustics, Rossing (1990) is important but it is a broad introduction to musical acoustics in its various practical settings. Fletcher and Rossing's *The Physics of Musical Instruments* (1998) focuses on instruments exclusively, but is a daunting work written primarily for physicists. Complexity aside, it is also not written in response to the needs and advanced practices of musicians. Writing on multiphonics the text states:

> Finally, we note that sometimes, in either brass or more particularly woodwind instruments, conditions may be modified by using special fingerings or lip positions so that the reed generator can drive, simultaneously, two horn resonances, w_1 and w_2, that are not harmonically related. The nonlinearity of the flow through the reed will then generate all sum and difference frequencies nw_1 +/- nw_2, to produce broadband multiphonic sound.

Such an understanding is essential, but given the modern comprehension and application of multiphonics, it is time for a conventional approach to defining multiphonics that moves directly into distinguishing groups of multiphonics—not simply at general levels like “collateral versus multi-driver,” or “flute collateral versus clarinet collateral,” but at quite specific and perceptually illuminating levels such as “types of flute collateral multiphonics versus types of clarinet collateral multiphonics.”

Common and more general definitions of multiphonics, like the one excerpted above, now feel like Jeans (1937) telling us in *Science & Music* that musical instruments under go a process of one-term nonlinear distortion whereby their fundamentals generate all upper harmonics. The process is important (as noted in Section B.2) but it must be immediately qualified to form a description that relates to our experience: clearly, what Jeans states, is not what is observed. The same is true of Fletcher and Rossing’s (1998) statement. Today we are interested in discussions of timbre that quickly address and account for obvious instrumental contrasts. Acoustic descriptions and explanations of multiphonics must also rise to this level if they are to evoke and illuminate the most basic forms we encounter when opening rich multiphonic catalogs like those by Dick (1975), Gallois (2009), Kientzy (1982), Levine (2002), Rehfeldt (1977), Shiung (2008) and Veal (1994).

B.6 Cognition and Waves: Another Look at Subjective & Objective Tones and Constructive & Destructive Interference

We have discussed various forms of common objective sum and difference tones (harmonic or inharmonic, one term or two-term, and produced at the level of the instrument, the ear, or by electronic sound modulating devices.) It goes without saying that microphones and speakers can produce such nonlinearities as well. There are no "ideal" speakers or microphones. And powerful inharmonic nonlinearly signals can distort microphones even in the ranges where they are expected to produce nearly flat responses. Therefore nonlinearities can appear at any point moving from instrument to microphone to speaker to listener. Once created they can be passed on linearly or receive further modification down the line

However, as mentioned earlier, it was long assumed that nonlinear sounds originated in the ear alone, and there was great debate (periodicity-detection versus pitch-detection theory) as to whether they had any physical basis. Accordingly, they were called subjective tones. Beyond this most anatomical and cognitive descriptions, whether supporting periodicity-detection or pitch-detection theory, tended to suggest the ear or brain calculated or provided these sounds to the listener *so that they could be heard*. However, as we have noted, current theory suggests that pitch detection is highly dependent on repetition rate and envelope repetition rate, which gives us a new way of looking both at the ear (Plack et al. 2005) and at the mind (Noë 2004).

Briefly, if a fundamental exists as a difference tone of two or more signals, but there is no energy for that fundamental emanating from the instrument, the ear does not need to provide it or calculate it—in any unusual sense—to perceive it. Through envelope repetition rate it can perceive it directly. In acoustics, envelope repetition rate is described as arising through constructive and destructive interference, which is a linear process. The ear can observe difference tones generated this way either as beats (if they are low frequency) or as fundamentals (if they are in the audio range). We say the ear doesn't need to do extra processing to observe these, not because it is passive, but because it only needs to process peak repetition rates, which is a process it is already enacting. No further calculation needs to be done. In much of the literature this direct approach to pitch recognition has been obscured by two factors. One is that the ear is liable to create the frequency of this low repetition rate directly as a distortion product. The other is the tendency to over imagine metaphorical frequency analysis procedures executed somewhere in the mind.

To clarify what makes our perception of sound different from that of frequency analysis consider this analogy. Frequency analysis reports to us the way a computer might report to us the motions of drawing a square with a pen plotter (see Figure B.3).

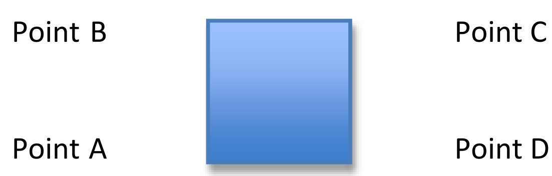

Figure B.3: Frequency analysis analogy: an action-based description of a square drawn by a pen plotter. The pen creates the square by moving from point A (bottom left) to point B (upper left) to point C (upper right) to point D (lower right) and back to point A.

There are precisely four movements, four energy components: movement from point A to B; from point B to C, from point C to D, and from point D to A. If we ask how many actions (what energy) and how many components (how many items) were contained in the process, the computer would tell us four. Those four lines were created and no further energy was used. No fifth or sixth object was created. This is crucial data if we want to reconstruct something—to know precisely what needs to be there. And that is what data frequency analysis provides. However, it would be odd to assume our eyes need to process those lines individually and then report back to us—*and thereby create the impression of*—a square. Noë's (2004) line of argument is that our eyes simply see the square; and that our mind can report back further analytical detail either through investigation, association, or by simple memory. (When we say: "all the sides are equal" have we checked? —have our minds specifically checked?)

In a similar way our ears observe the items *and* the total pattern—which in this case is the envelope repetition rate. Our ears can "see the square." Therefore, given the complex waveform ($f1$ = 300 Hz, $f2$ = 400 Hz, $f3$ = 500 Hz, $f4$ = 600 Hz) the ear also observes peak envelope repetition rate of $f0$ = 100 Hz. It doesn't need to deduce this. If a nonlinear distortion is also created at 100 Hz, it is because of a redundancy built into the system itself, it is not a required cognitive courtesy needed for us to assemble the sound.

This approach also explains why subjective sum tones are so difficult to hear. As an artifact of constructive and destructive interference sum tones are usually utterly opaque. If the highest peaks in two signals are 300 Hz and 350 Hz, and those waves combine linearly via constructive and destructive interference, the fast peaks will align six out of seven times. This provides excellent constructive interference for the envelope repetition rate of 50 Hz, but what of 650 Hz? If the peaks combined 50 times, we only have 600 separate peaks; and they are not periodic. Even if the peaks were prime, they would not form a stable repetition rate. Consider this analogy: does "4 against 5" sound like eight 32nds? (Of course, this is an analogy only, and not meant to suggest a direct correlate between how we hear rhythm and pitch.) Further, these peaks *can't* combine if we are using them to create a high "sum" number, therefore they are almost always interfered with destructively: they can not get consistent amplitude gain via constructive interference—unless the sound itself has a *higher harmonic of the target sum tone itself* for the peaks to combine with. (Think $f1$ = 43 Hz; $f2$ = 53 Hz; $f3$ = 192 Hz, with $f1 + f2$ creating the quasi-sum tone 96 Hz, which can constructively interfere with every other peak in $f3$.) This is not to say that non-nonlinear sum tones don't exist. (Nonlinearities arise through a different process, but their amplitude energies would have similar challenges, unless perhaps they coincided with a massive formant frequency.) Rather the point is to suggest that it is helpful to think about the salience of waves themselves, rather than debating whether or whether not a mathematical operation is taking place. And for musicians dealing with complex and opaque multiphonics the larger point is, if you want the brain to notice something, make it high profile.